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A Stochastic Expected Utility Theory

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Abstract:

This paper proposes a new model that explains the violations of expected utility theory through the role of random errors. The paper analyzes decision making under risk when individuals make random errors when they compute expected utilities. Errors are drawn from the normal distribution, which is truncated so that the stochastic utility of a lottery cannot be greater (lower) than the utility of the highest (lowest) possible outcome. The standard deviation of random errors is higher for lotteries with a wider range of possible outcomes. It converges to zero for lotteries converging to a degenerate lottery. The model explains all major stylized empirical facts such as the Allais paradox and the fourfold pattern of risk attitudes. The model fits the data from ten well-known experimental studies at least as good as cumulative prospect theory.

Keywords: decision theory, stochastic utility, expected utility theory, cumulative prospect theory

JEL Classification codes: C91, D81

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A stochastic expected utility theory

> "Perhaps we should now spend some time on thinking about the noise, rather than about even more alternatives to EU?"
> Hey and Orme (1994), *Econometrica* **62**, p.1322

1. Introduction

Expected utility theory or EUT (e.g. von Neumann and Morgenstern, 1944) is a compelling normative decision theory in choice under risk (e.g. Knight, 1921). However, persistent violations of EUT, such as the Allais paradox (e.g. Allais, 1953), make EUT a descriptively inadequate theory (e.g. Camerer, 1995). Many theories have been proposed to improve the descriptive fit of EUT by introducing a few extra parameters and relaxing some of EUT axioms (see Starmer, 2000, for a review). Though no clear successor of EUT emerged (e.g. Harless and Camerer, 1994; Hey and Orme, 1994), a decision theory that explains the largest fraction of the empirical evidence appears to be cumulative prospect theory or CPT (e.g. Tversky and Kahneman, 1992).

This paper takes a new approach to explain the violations of EUT through the role of random errors. Camerer (1989), Starmer and Sugden (1989) and Wu (1994) provide extensive experimental evidence that there is some degree of randomness in the observed individual preference between lotteries. In a repeated choice between the same two lotteries (with a possibility of indifference) individuals make identical decisions only in around 75% of all cases (e.g. Hey and Orme, 1994). In other words, decision making under risk is inherently stochastic. The model presented in this paper describes the decisions of individuals who make random errors when they compute the expected utility of a lottery. The reexamination of experimental evidence reveals that the predictive power of this simple model is at least as good as that of CPT.

The remainder of this paper is organized as follows. A stochastic expected utility theory or StEUT is described in section 2. Section 3 demonstrates how StEUT explains all major stylized empirical facts such as the Allais paradox and the fourfold pattern of risk attitudes (e.g. Tversky and Kahneman, 1992). StEUT and CPT are contested against each other in section 4 using the data from ten well-known experimental studies. Section 5 concludes.

2. Theory

Let $L(x_1, p_1; ... x_n, p_n)$ denote a lottery that delivers an outcome x_i with probability p_i, $i \in [1, n]$. Furthermore, let x_1 (x_n) be the lowest (highest) outcome such that $p_1 \neq 0$ ($p_n \neq 0$). The stochastic expected utility of lottery $L(x_1, p_1; ... x_n, p_n)$ is given by equation (1).

$$U(L) = \sum_{i=1}^{n} p_i u(x_i) + \xi_L \qquad (1)$$

Utility function $u : \mathrm{R} \to \mathrm{R}$ is defined over changes in wealth rather than absolute wealth levels, as proposed by Markowitz (1952). An error term ξ_L is independently and normally distributed with a zero mean. Although the distribution of individual random errors is unknown, the distribution of random errors averaged across the subjects converges to the normal distribution due to the Central Limit Theorem. Thus, at least for the aggregate data, the normal distribution is a reasonably justifiable assumption. The specification of an error term (1) also appears in Hey and Orme (1994, p.1301) and Gonzalez and Wu (1999).

The stochastic expected utility of a lottery cannot be less than the utility of the lowest possible outcome (see, however, Gneezy et al., 2004). Similarly, it cannot exceed the utility of the highest possible outcome. Therefore, the normal distribution of an error term is truncated so that $u(x_1) \leq \sum_{i=1}^{n} p_i u(x_i) + \xi_L \leq u(x_n)$. Specifically, the probability density function of an error term ξ_L is given by equation (2).

$$prob(\xi_L = v) = \begin{cases} 0, & v < u(x_1) - \mu_L \\ \dfrac{1}{\Phi(u(x_n) - \mu_L) - \Phi(u(x_1) - \mu_L)} \dfrac{e^{-\frac{v^2}{2\sigma_L^2}}}{\sigma_L \sqrt{2\pi}}, & u(x_1) - \mu_L \leq v \leq u(x_n) - \mu_L \\ 0, & v > u(x_n) - \mu_L \end{cases} \quad (2)$$

where $\mu_L = \sum_{i=1}^{n} p_i u(x_i)$ is the expected utility of a lottery and $\Phi(.)$ is the cumulative density function of the normal distribution with zero mean and standard deviation σ_L.

The standard deviation of random errors σ_L has two properties. First, ceteris paribus, the standard deviation of random errors is higher for lotteries with a wider range of possible outcomes. Second, the standard deviation of random errors converges to zero for lotteries converging to a degenerate lottery, i.e. $\lim_{p_i \to 1} \sigma_L = 0, \forall i \in [1, n]$. The stochastic expected utility of a degenerate lottery that delivers an outcome x_i for certain is uniquely determined as $u(x_i)$. In this case the stochastic component ξ_L in equation (1) disappears. Two properties of the standard deviation of random errors together with equations (1)-(2) complete the description of StEUT. Obviously, when the standard deviation of random errors is zero for all lotteries, StEUT coincides with the deterministic EUT.

3. Explanation of the stylized facts

Having described the building blocks of StEUT, this section explores how this simple model explains theoretically the major stylized empirical facts. Subsection 3.1 below provides intuition behind the StEUT explanation of the most famous example of EUT violations—the Allais paradox. Subsections 3.2, 3.3, 3.5 and 3.5 present technical results demonstrating that StEUT predicts correspondingly the fourfold pattern of risk attitudes, the generalized common consequence effect (of which the Allais paradox is one specific example), the common ratio effect and the violation of betwenness.

3.1. The Allais paradox

The Allais paradox refers to the choice pattern $L_1(10^6,1) \succ L_2(0,0.01;10^6,0.89;5\cdot10^6,0.1)$ and $L_2'(0,0.9;5\cdot10^6,0.1) \succ L_1'(0,0.89;10^6,0.11)$, which is frequently found in the empirical studies (e.g. Slovic and Tversky, 1974). Deterministic EUT cannot explain this choice pattern (e.g. Allais, 1953). Figure 1 demonstrates how StEUT explains the Allais paradox. The horizontal axis is the subjective utility scale ranging from $u(0)$ to $u(5\cdot10^6)$. The utility of lottery L_1 is deterministic and it is uniquely determined as $u(10^6)$. It is assumed that $u(10^6)-u(0) > u(5\cdot10^6)-u(10^6)$. The stochastic expected utility of lottery L_2 is represented on figure 1 by a solid probability density function (PDF) of the normal distribution with mean $0.89\cdot u(10^6)+0.1\cdot u(5\cdot10^6)$. Similarly, $U(L_1')$ is depicted by a doted PDF of the normal distribution with mean $0.11\cdot u(10^6)$ and $U(L_2')$—by a dashed PDF of the normal distribution with mean $0.1\cdot u(5\cdot10^6)$. For demonstration purposes, it is assumed that $0.1\cdot u(5\cdot10^6) > 0.11\cdot u(10^6)$, i.e. the mode of the dashed distribution is greater than the mode of the dotted distribution and the mode of the solid distribution is greater that $u(10^6)$.

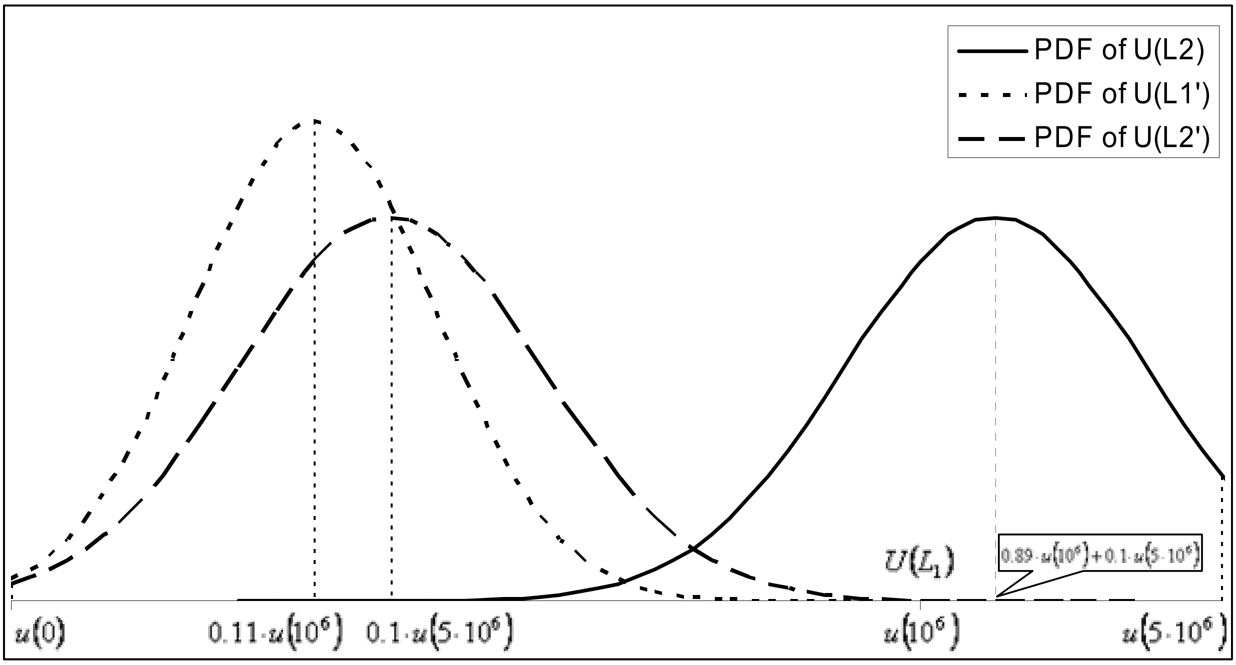

Figure 1 StEUT and the Allais paradox

The distributions of $U(L_2)$ and $U(L_2')$ have the same standard deviation because L_2 and L_2' have the same range of possible outcomes (from zero to five million). However, the distribution of $U(L_1')$ has a lower standard deviation because lottery L_1' has a narrower range of possible outcomes (from zero to one million). For lotteries L_2, L_1' and L_2', the highest probability attached to an outcome is correspondingly 0.89, 0.89 and 0.9. These probabilities are nearly identical. Thus, if the variance of random errors decreases because lotteries L_2, L_1' and L_2' approach to a degenerate lottery, this effect is nearly identical for all three lotteries.

The distribution of $U(L_2)$ is truncated more severely from the upper bound than from the lower bound because its mode $u(10^6)$ is located closer to the upper bound $u(5 \cdot 10^6)$ than to the lower bound $u(0)$. Thus, for lottery L_2 the realization of a random error is more likely to decrease its utility rather than increase it. Although the mode of the distribution of $U(L_2)$ is greater than $u(10^6)$, the mean of the distribution of $U(L_2)$ can be lower than $u(10^6)$. In other words, lottery L_1 may be preferred to lottery L_2 because random errors are more likely to undervalue the utility of L_2 rather than overvalue it.

The distributions of $U(L_1')$ and $U(L_2')$ are truncated more severely from the lower bound than from the upper bound because their corresponding modes $0.11 \cdot u(10^6)$ and $0.1 \cdot u(5 \cdot 10^6)$ are located closer to the lower bound $u(0)$ than to the upper bound $u(5 \cdot 10^6)$. For lotteries L_1' and L_2' the realization of a random error is more likely to increase rather than decrease their utilities. Due to truncation, the distribution of $U(L_1')$ loses a longer lower tail then the distribution of $U(L_2')$. However, the distribution of $U(L_1')$ has a smaller standard deviation, i.e. its tails are thinner. An overall relative effect is ambiguous. Random errors can overvalue the utility of L_1' to a greater extent than L_2' (which is compatible with the Allais paradox) and vise versa (which may refute the Allais paradox). Thus, L_2' may be preferred to L_1' when simultaneously L_1 is preferred to L_2.

Notice, that the occurrence of the Allais paradox can be reduced if lottery L_1 delivers almost one million (not with certainty). In this case the utility of L_1 is no longer deterministic and it becomes affected by random errors. Similarly as for lottery L_2, random errors are more likely to undervalue the utility of L_1 rather than overvalue it. Thus, for lotteries L_1 and L_2 the relative impact of random errors becomes of a second order and the Allais paradox can disappear. Conlisk (1989) provides empirical evidence supporting this prediction: the Allais paradox disappears when lotteries involved in the paradox are shifted inside the probability triangle (e.g. Marschak, 1950; Machina, 1982). Subsection 4.8 below discusses how StEUT fits the actual experimental evidence reported in Conlisk (1989)

3.2. The fourfold pattern of risk attitudes

The fourfold pattern of risk attitudes refers to an empirical observation that individuals often exhibit risk aversion when dealing with probable gains or improbable losses. The same individuals often exhibit risk seeking when dealing with improbable gains or probable losses (e.g. Tversky and Kahneman, 1992). One of the implications of the fourfold pattern of risk attitudes is that individuals can simultaneously purchase insurance and public lottery tickets. The latter paradoxical observation was the first descriptive challenge for the deterministic EUT (e.g. Friedman and Savage, 1948).

To demonstrate how StEUT explains the fourfold pattern of risk attitudes, it is necessary first to calculate the certainty equivalent (or subjective price) of an arbitrary lottery L according to StEUT. By definition, the certainty equivalent is such an outcome CE that $u(CE)=E(U(L))$. Using equation (1) this definition can be rewritten as equation (3).

$$CE = u^{-1}\left(\sum_{i=1}^{n} p_i u(x_i) + E(\xi_L)\right) = u^{-1}\left(\sum_{i=1}^{n} p_i u(x_i) + \int_{-\infty}^{+\infty} v \cdot prob(\xi_L = v) dv\right) \qquad (3)$$

Inserting equation (2) into (3) and simplifying the algebra, we obtain equation (4).

$$CE = u^{-1}\left(\mu_L + \frac{\sigma_L}{\sqrt{2\pi}} \frac{e^{-\frac{(u(x_1)-\mu_L)^2}{2\sigma_L^2}} - e^{-\frac{(u(x_n)-\mu_L)^2}{2\sigma_L^2}}}{\Phi(u(x_n)-\mu_L)-\Phi(u(x_1)-\mu_L)}\right) \qquad (4)$$

Let $CE\big|_{\sigma_L=0} = u^{-1}(\mu_L) = u^{-1}\left(\sum_{i=1}^{n} p_i u(x_i)\right)$ denote the certainty equivalent of lottery L according to the deterministic EUT. Equation (4) then implies that $\lim_{\mu_L \to u(x_1)} CE > CE\big|_{\sigma_L=0}$ and $\lim_{\mu_L \to u(x_n)} CE < CE\big|_{\sigma_L=0}$.[1] In other words, random errors overvalue the certainty equivalent (4) of a lottery whose expected utility μ_L is close to the utility of the lowest possible outcome $u(x_1)$. At the same time, random errors undervalue the certainty equivalent (4) of a lottery whose expected utility is close to the utility of the highest possible outcome $u(x_n)$.

If utility function $u(.)$ is concave then $CE\big|_{\sigma_L=0} = u^{-1}\left(\sum_{i=1}^{n} p_i u(x_i)\right) < \sum_{i=1}^{n} p_i x_i$. In other words, an individual with concave utility function is always risk averse according to the deterministic EUT. According to StEUT, an individual with concave utility function is always risk averse when the expected utility of a lottery is close to the utility of the highest possible

[1] This result follows directly from equation (4). The difference $\Phi(u(x_n)-\mu_L)-\Phi(u(x_1)-\mu_L)$ is always greater than zero and the difference $e^{-\frac{(u(x_1)-\mu_L)^2}{2\sigma_L^2}} - e^{-\frac{(u(x_n)-\mu_L)^2}{2\sigma_L^2}}$ is greater (lower) than zero when $\mu_L = u(x_1)$ ($\mu_L = u(x_n)$).

outcome. In this case, $\lim_{\mu_L \to u(x_n)} CE < CE\big|_{\sigma_L=0} < \sum_{i=1}^{n} p_i x_i$. When the expected utility of a lottery is close to the utility of the lowest possible outcome, i.e. $\mu_L \to u(x_1)$ the prediction of StEUT is ambiguous. An individual with concave utility function may be risk seeking when random errors overvalue the certainty equivalent (4) so that $CE > \sum_{i=1}^{n} p_i x_i$ although $CE\big|_{\sigma_L=0} < \sum_{i=1}^{n} p_i x_i$.

Similarly, according to the deterministic EUT an individual with convex utility function is always risk seeking. According to StEUT an individual with convex utility function is always risk seeking when the expected utility of a lottery is close to the utility of the lowest possible outcome. He or she may be risk averse when the expected utility of a lottery is close to the utility of the highest possible outcome. Risk aversion occurs when random errors undervalue the certainty equivalent of a lottery below its expected value though in the absence of any error the certainty equivalent of a lottery is above its expected value.

To summarize, both for concave and convex subjective utility functions StEUT admits the incidence of risk seeking (risk aversion) for lotteries whose expected utility is close to the utility of the lowest (highest) possible outcome. In the terminology of Tversky and Kahneman (1992) a lottery whose expected utility is close to the utility of the lowest possible outcome is called either an improbable gain or a probable loss. Similarly, a lottery whose expected utility is close to the utility of the highest possible outcome is called either a probable gain or an improbable loss. Therefore, StEUT is able to explain the fourfold pattern of risk attitudes. Subsection 4.1 below discusses how StEUT fits the actual experimental evidence of the fourfold pattern of risk attitudes reported in Tversky and Kahneman (1992).

3.3. Generalized common consequence effect

The common consequence effect is the following empirical observation. There exist pairs of lotteries $S(x_1,1-p-r;x_2,p+r)$ and $R(x_1,1-q-r;x_{2,}r;x_3,q)$, such that an individual prefers *S* to *R* if $r=1-p$ and the same individual prefer *R* to *S* if $r=0$ (e.g. Slovic and Tversky, 1974; MacCrimmon and Larsson, 1979). The Allais paradox is a special case of the common consequence effect when $x_1=0, x_2=10^6, x_3=5\cdot 10^6, p=0.11$ and $q=0.1$ (e.g. Starmer, 2000).

The generalized common consequence effect refers to the following two empirical observations. First, there exist lotteries *S* and *R* with probabilities *p* and *q* close to zero such that when probability *r* decreases from $1-p$ to zero the fraction of individuals who prefer *S* over *R* first decreases and then increases (e.g. Wu and Gonzalez, 1996). Second, there exist lotteries $S'(x_1,t-p;x_2,p+r;x_3,1-t-r)$ and $R'(x_1,1-q;x_2,r;x_3,q-r)$ with probabilities p and $1-q$ close to zero such that when probability *r* decreases from $1-t$ to zero the fraction of individuals who prefer *S* over *R* first increases and then decreases (e.g. Wu and Gonzalez, 1998). Intuitively, when the probability mass is shifted from the medium outcome to the lowest possible outcome, the choice of a riskier lottery becomes first more likely and then—less likely. This pattern is reverse when the probability mass is shifted from the medium outcome to the highest outcome.

To demonstrate how StEUT explains the common consequence effect, it is necessary first to calculate the probability of choosing lottery *S* over *R* in a binary choice. According to equation (1), $S \succ R \Leftrightarrow \mu_S+\xi_S>\mu_R+\xi_R$. Thus, $prob(S\succ R)=\int_{-\infty}^{+\infty} prob(\xi=v)\cdot prob(\xi_R<v+\mu_S-\mu_R)dv$.

The last equation becomes (5) when we substitute for the probability density function from (2).

$$prob(S\succ R)=\int_{u(x_1^S)-\mu_S}^{u(x_n^S)-\mu_S} \frac{e^{-\frac{v^2}{2\sigma_S^2}}}{\sigma_S\sqrt{2\pi}} \frac{\Phi_R\left(\min\{v+\mu_S-\mu_R,u(x_n^R)-\mu_R\}\right)-\Phi_R\left(\min\{v+\mu_S-\mu_R,u(x_1^R)-\mu_R\}\right)}{\left(\Phi_S\left(u(x_n^S)-\mu_S\right)-\Phi_S\left(u(x_1^S)-\mu_S\right)\right)\left(\Phi_R\left(u(x_n^R)-\mu_R\right)-\Phi_R\left(u(x_1^R)-\mu_R\right)\right)}dv \quad (5)$$

where Φ_L is the cumulative density function of the normal distribution with zero mean and a standard deviation σ_L and x_1^L (x_n^L) is the lowest (highest) possible outcome of lottery $L \in \{S, R\}$.

The lotteries involved in the generalized common consequence effect are constructed in such a way that the range of possible outcomes of one lottery (S and S') is always within the range of possible outcomes of the other lottery (R and R'). Therefore, when $v \in \left[u\left(x_1^S\right) - \mu_S, u\left(x_n^S\right) - \mu_S\right]$ we can infer that $\min\left\{v + \mu_S - \mu_R, u\left(x_n^R\right) - \mu_R\right\} = v + \mu_S - \mu_R$ and $\min\left\{v + \mu_S - \mu_R, u\left(x_1^R\right) - \mu_R\right\} = u\left(x_1^R\right) - \mu_R$. Plugging these results into equation (5) and simplifying algebra yields equation (6).

$$prob(S \succ R) = \frac{1}{\Phi_R\left(u\left(x_n^R\right) - \mu_R\right) - \Phi_R\left(u\left(x_1^R\right) - \mu_R\right)} \left(\frac{\displaystyle\int_{u\left(x_1^S\right) - \mu_S}^{u\left(x_n^S\right) - \mu_S} \frac{e^{-\frac{v^2}{2\sigma_S^2}}}{\sigma_S \sqrt{2\pi}} \Phi_R\left(v + \mu_S - \mu_R\right) dv}{\Phi_S\left(u\left(x_n^S\right) - \mu_S\right) - \Phi_S\left(u\left(x_1^S\right) - \mu_S\right)} - \Phi_R\left(u\left(x_1^R\right) - \mu_R\right) \right) \quad (6)$$

Consider now how the shift of the probability mass from the medium to the lowest possible outcome affects $prob(S \succ R)$ given by equation (6). In the simplest case when *S* and *R* have the same expected utility ($\mu_S = \mu_R$) and the same standard deviation of random errors ($\sigma_S = \sigma_R$), equation (6) implies that $prob(S \succ R) = \frac{1}{2} \frac{\Phi_R\left(u\left(x_2\right) - \mu_R\right) - \Phi_R\left(u\left(x_1\right) - \mu_R\right)}{\Phi_R\left(u\left(x_3\right) - \mu_R\right) - \Phi_R\left(u\left(x_1\right) - \mu_R\right)} < \frac{1}{2}$. Therefore, an individual prefers *R* to *S* when $\mu_S = \mu_R$ and $\sigma_S = \sigma_R$. Notice that lottery *R* has a wider range of possible outcomes than lottery *S*. According to the assumption of StEUT, this implies that ceteris paribus $\sigma_S < \sigma_R$. However, when the probability mass is shifted to the lowest possible outcome, *R* converges to a degenerate lottery (given that a fixed probability *q* attached to the highest outcome is small). According to another assumption of StEUT, σ_R then converges to zero. Therefore, we may observe that $\sigma_S = \sigma_R$ only when the probability mass is shifted to the lowest possible outcome. All together this implies that StEUT predicts preference $R \succ S$ when *R* and *S* have the same expected utility and the probability mass is shifted to the lowest outcome (so that $\sigma_S = \sigma_R$).

When *R* and *S* have the same expected utility ($\mu_S = \mu_R$) and the probability mass is shifted to the medium outcome, it must be the case that $\sigma_S < \sigma_R$ because *R* has a wider range of possible outcomes and, moreover, *S* converges to a degenerate lottery, i.e. σ_S converges to zero. When the probability mass is shifted to the medium outcome, the expected utility of *S* converges to the utility of the medium outcome, which is simultaneously the highest possible outcome of lottery *S*, i.e. $\mu_S \to u\left(x_n^S\right)$. It is always possible to find μ_S, sufficiently close to $u\left(x_n^S\right)$, such that equation (7) holds.

$$\int_{u\left(x_1^S\right)-\mu_S}^{u\left(x_n^S\right)-\mu_S} \frac{e^{-\frac{v^2}{2\sigma_S^2}}}{\sigma_S\sqrt{2\pi}} \Phi_R\left(v+\mu_S-\mu_R\right)dv \geq \int_{u\left(x_1^S\right)-\mu_S}^{u\left(x_n^S\right)-\mu_S} \frac{e^{-\frac{v^2}{2\sigma_S^2}}}{\sigma_S\sqrt{2\pi}} \Phi_S(v)dv = \frac{1}{2}\Phi_S^2\left(u\left(x_n^S\right)-\mu_S\right)-\frac{1}{2}\Phi_S^2\left(u\left(x_1^S\right)-\mu_S\right) \quad (7)$$

The intuition is the following. When $\mu_S \to u\left(x_n^S\right)$ the integration is conducted primarily over the interval in which $v<0$. For $v<0$ the cumulative density function $\Phi_R(v)$ with a higher standard deviation is located above the cumulative density function $\Phi_S(v)$ with a lower standard deviation, i.e. $\Phi_R(v) > \Phi_S(v)$.

Plugging result (7) into equation (6) and simplifying the algebra yields $prob(S \succ R) \geq \frac{1}{2}\frac{\Phi_S\left(u\left(x_n^S\right)-\mu_S\right)+\Phi_S\left(u\left(x_1^S\right)-\mu_S\right)-2\Phi_R\left(u(x_1)-\mu_R\right)}{\Phi_R\left(u(x_3)-\mu_R\right)-\Phi_R\left(u(x_1)-\mu_R\right)}$. Thus, $prob(S \succ R) > 0.5$, i.e. an individual prefers *S* to *R* if and only if inequality (8) holds.

$$\Phi_S\left(u\left(x_n^S\right)-\mu_S\right)+\Phi_S\left(u\left(x_1^S\right)-\mu_S\right) > \Phi_R\left(u(x_3)-\mu_R\right)+\Phi_R\left(u(x_1)-\mu_R\right) \quad (8)$$

When $\mu_R > (u(x_3)+u(x_1))/2$ the right hand side of (8) is always less than unity. However, when σ_S converges to zero the left hand side of (8) converges to unity. In this case inequality (8) would always hold. Therefore, StEUT predicts preference $S \succ R$ when *R* and *S* have the same expected utility $\mu_S = \mu_R > (u(x_3)+u(x_1))/2$ and the probability mass is shifted to the medium outcome (so that $\sigma_S \to 0$). In a degenerate case when lottery *S* delivers the medium outcome x_2 for certain ($r = 1-p$), we have $prob(S \succ R) = \dfrac{\Phi_R(u(x_2)-\mu_R)-\Phi_R(u(x_1)-\mu_R)}{\Phi_R(u(x_3)-\mu_R)-\Phi_R(u(x_1)-\mu_R)}$ and *S* is preferred to *R* if and only if $\Phi_R(u(x_2)-\mu_R) > (\Phi_R(u(x_3)-\mu_R)+\Phi_R(u(x_1)-\mu_R))/2$.

Notice that when the probability mass is shifted to the lowest possible outcome so that lottery *R* converges to a degenerate lottery (probability q is small), σ_S converges to zero. In this case it is possible that $\sigma_R < \sigma_S$. Additionally, when the probability mass is shifted to the lowest possible outcome the expected utility of *S* converges to the utility of the lowest outcome, i.e. $\mu_S \to u(x_1^S)$. Thus, in the right hand side of equation (6) the integration is conducted primarily over the interval in which $v > 0$. For $v > 0$ the cumulative density function $\Phi_R(v)$ with a lower standard deviation is located above the cumulative density function $\Phi_S(v)$ with a higher standard deviation, i.e. $\Phi_R(v) > \Phi_S(v)$. Therefore, $prob(S \succ R)$ when $\sigma_R < \sigma_S$ is higher than $prob(S \succ R)$ when $\sigma_R = \sigma_S$, and an individual may prefer *S* to *R*. To summarize, when lotteries *S* and *R* have the same expected utility and the probability mass is shifted from the medium to the lowest outcome, StEUT predicts that $prob(S \succ R)$ first decreases, but then it may increase.

Intuitively, when the probability mass is allocated to the medium outcome, which is close to the highest possible outcome in terms of individual's utility, an individual prefers *S* to *R*. Utility of *S* is deterministic (not affected by random errors). However, random errors are likely to undervalue the utility of *R*. When the probability mass is allocated to the lowest possible outcome, random errors are likely to overvalue the utility of both *S* and *R*. This effect is stronger for *R*, i.e. an individual prefers *R* to *S*, because *R* has a wider range of possible outcomes (and hence a higher volatility of errors). However, when the probability mass is shifted to the lowest outcome so dramatically that *R* delivers the lowest outcome almost for certain, an individual may prefer *S* to *R*. The variance of errors that distort the utility of *R* converges to zero, i.e. random errors almost do not overvalue the utility of *R*, but they do overvalue the utility of *S*.

Notice that according to StEUT, the incidence of the common consequence effect decreases if lottery *S* has the same range of possible outcomes as lottery *R*. Camerer (1992) finds experimental evidence confirming this prediction: when *S* and *R* are located inside the probability triangle the common consequence effect largely disappears (section 4.6 below).

Consider now the situation when the probability mass is shifted from the medium outcome to the highest outcome. Lotteries S' and R' have the same range of possible outcomes, i.e. ceteris paribus $\sigma_S = \sigma_R$. When the probability mass is allocated to the medium outcome ($r = 1 - p$), lottery S' converges to a degenerate lottery, i.e. $\sigma_S \to 0$, and hence $\sigma_S < \sigma_R$. When the probability mass is allocated to the highest outcome ($r = 0$), lottery R' converges to a degenerate lottery and hence $\sigma_R < \sigma_S$.

In the simplest case when $\mu_S = \mu_R$ and $\sigma_S = \sigma_R$, equation (6) implies that $prob(S \succ R) = 0.5$. When $\sigma_S < \sigma_R$ and $\mu_{S'} - u(x_1^{S'}) < u(x_n^{S'}) - \mu_{S'}$, equation (6) implies that $prob(S \succ R) < 0.5$, i.e. an individual prefers *R* over *S*. Thus, when the probability mass is shifted to the medium outcome, StEUT predicts that $prob(S \succ R)$ decreases if $\mu_{S'} < \left(u(x_n^{S'}) + u(x_1^{S'})\right)/2$. When the probability mass is shifted to the highest outcome, i.e. $\mu_{S'} \to u(x_n^{S'})$, it was already established above that $\sigma_R < \sigma_S$. Equation (6) then implies that $prob(S \succ R) < 0.5$, i.e. an individual prefers *R* over *S*. To summarize, when lotteries *S* and *R* have the same expected utility and the probability mass is shifted from the medium to the highest outcome, StEUT predicts that $prob(S \succ R)$ may first increase but then it decreases. Thus, theoretically, StEUT is able to explain the generalized common consequence effect. Subsection 4.3 below discusses how StEUT accommodates the actual experimental evidence of the generalized common consequence effect reported in Wu and Gonzalez (1996).

3.4. Common ratio effect

The common ratio effect is an empirical observation that there exist pairs of lotteries $S(x_1, 1-r; x_2, r)$ and $R(x_1, 1-\theta r;; x_3, \theta r)$, $x_2 < x_3$ and $0 < \theta < 1$, such that an individual prefers *S* to *R* when *r* is close to unity and the same individual prefers *R* to *S* when *r* is close to zero (e.g. Starmer, 2000). According to StEUT, the probability that an individual chooses *S* over *R* is given by equation (6). For simplicity, utility function is normalized so that $u(x_1) = 0$ and $u(x_3) = 0$. Equation (6) then becomes equation (9) for a pair of lotteries involved in the common ratio effect.

$$prob(S \succ R) = \frac{1}{\Phi_R(1-\theta r) - \Phi_R(-\theta r)} \left(\frac{\displaystyle\int_{-ru(x_2)}^{(1-r)u(x_2)} \frac{e^{-\frac{v^2}{2\sigma_S^2}}}{\sigma_S \sqrt{2\pi}} \Phi_R\big(v + r(u(x_2) - \theta)\big) dv}{\Phi_S\big((1-r)u(x_2)\big) - \Phi_S\big(-ru(x_2)\big)} - \Phi_R(-\theta r) \right) \quad (9)$$

Assume that $\sigma_S = \sigma_R$ when *r* is converges to zero. This assumption is justifiable because although *R* has a wider range of possible outcomes, i.e. ceteris paribus $\sigma_S < \sigma_R$, *R* also converges to a degenerate lottery faster than does *S*. It follows then from equation (10) that

$\lim_{r\to 0} prob(S \succ R) = \frac{1}{2}\lim_{r\to 0}\frac{\Phi_R(u(x_2)) - \Phi_R(0)}{\Phi_R(1) - \Phi_R(0)} \leq \frac{1}{2}$, and this result does not depend on any assumptions about $prob(S \succ R)$ when r is close to unity. In other words, according to StEUT an individual prefers R over S when r is close to zero (so that $\sigma_S = \sigma_R$), and the same individual may prefer S to R when r is close to unity. Subsections 4.5 and 4.9 below discuss how StEUT fits the actual experimental evidence of the common ratio effect reported correspondingly in Bernasconi (1994) and Loomes and Sugden (1998).

3.5. Violation of betweenness

The betweenness axiom states that if an individual is indifferent between two lotteries than a probability mixture of these two lotteries is equally good (e.g., Dekel, 1986). Empirical studies documented systematic violations of betwenness (e.g. Coombs and Huang, 1976; Chew and Waller, 1986; Battalio et al., 1990; Prelec, 1990; Gigliotti and Sopher, 1993; Camerer and Ho, 1994). A typical empirical test of betwenness is the following. For two arbitrary lotteries S and R and a probability mixture $M = \theta \cdot S + (1-\theta)\cdot R$, $\theta \in (0,1)$, an individual is asked to choose one lottery from three sets $\{S, R\}$, $\{S, M\}$ and $\{M, R\}$. Eight choice patterns are possible theoretically. Only two of them are consistent with betwenness: $S \succ R$, $S \succ M$, $M \succ R$, denoted for simplicity as *SSM*, and *RMR*. Choice patterns *SMM* and *RMM* indicate that an individual likes randomization, i.e. they reveal quasi-concave preferences. Choice patterns *SSR* and *RSR* indicate that an individual dislikes randomization, i.e. they reveal quasi-convex preferences. An asymmetric split between quasi-concave and quasi-convex preferences is taken as an evidence of betwenness violation.

The betwenness axiom is a weaker version of the independence axiom of the deterministic EUT (e.g., Dekel, 1986). Thus, deterministic EUT cannot explain the violation of betwenness. StEUT is able to explain such violation as it is demonstrated by one specific example below. Consider lotteries *S* and *R* such that *S* and *R* have the same lowest possible outcome, but the highest possible outcome of *S* is lower than the highest possible outcome of *R*. According to the assumption of StEUT, this implies that $\sigma_S < \sigma_R$ because *R* has a wider range of possible outcomes. Let the expected utility of *S* be closer to the utility of the lowest outcome than to the utility of the highest outcome. Finally, assume for the sake of argument that $prob(S \succ R) > 0.5$, i.e. an individual prefers *S* and *R*. Empirical tests of betwenness employing lotteries *S* and *R* that satisfy the above assumptions were conducted inter alia by Prelec (1990), Camerer and Ho (1994) and Bernasconi (1994).

When $\theta \to 1$ a probability mixture *M* converges to lottery *S*. In this case *S* and *M* have similar expected utilities ($\mu_S \cong \mu_M$), but $\sigma_S < \sigma_M$ because *M* has a wider range of possible outcomes. $prob(S \succ M)$ is determined by equation (6). Taking into consideration that μ_S is as close as necessary to $u(x_1^S)$ by assumption and following the same argument as described in section 3.3 above, it is possible to prove that $prob(S \succ M) < 0.5$ if $\mu_M < \left(u(x_n^M) + u(x_1^M)\right)/2$. In other words, an individual may prefer *M* to *S* although at the same time *S* is preferred to *R*, which is a violation of betwenness (e.g. Prelec, 1990).

Intuitively, *S* and *M* have almost identical expected utilities, which are close to the utility of their lowest possible outcome. In this case, random errors are likely to overvalue the stochastic utility of both *S* and *M*, but this effect is stronger for *M* because the stochastic utility of *M* has a higher volatility of random errors. Thus, an individual may prefer *M* to *S* and *S* to *R*. In other words, StEUT predicts a higher incidence of the quasi-concave preferences. Subsections 4.4 and 4.5 below discuss how StEUT fits the actual experimental evidence of betwenness violation reported correspondingly in Camerer and Ho (1994) and Bernasconi (1994).

4. Fit to experimental data

The previous section demonstrated that StEUT is able to explain theoretically all major well-known empirical facts. This section explores how StEUT fits the actual empirical data. In the following subsections 4.1.-4.9 the same parsimonious parametric form of StEUT is used to accommodate the aggregate choice pattern revealed in nine well-known experimental studies. Of course, individuals do not share identical preferences. However, a single-agent stochastic model is a simple method for integrating data from many studies, where individual estimates have low power, e.g. when one subject makes only a few decisions (e.g. Camerer and Ho, 1994, p.186). Such approach is also relevant in an economic sense because it describes the behavior of a "representative agent" (e.g. Wu and Gonzalez, 1996). In the subsection 4.10, the parametric form of StEUT is estimated separately for every subject using the experimental data from Hey and Orme (1994).

4.1. Tversky and Kahneman (1992)

Tversky and Kahneman (1992) elicited the certainty equivalents of 56 lotteries and found strong empirical support for the fourfold pattern of risk attitudes. The first column of table 1 presents 28 lotteries from Tversky and Kahneman (1992) that have only positive outcomes. The second column of table 1 presents a median certainty equivalent for every lottery, as elicited in the experiment (data are taken from table 3 in Tversky and Kahneman (1992), p.307). For comparison, the third column of table 1 presents the certainty equivalents predicted by CPT with the best fitting parameters ($\alpha = 0.88, \gamma = 0.61$) estimated by Tversky and Kahneman (1992). Following the same format, table 2 presents the results of Tversky and Kahneman (1992) for 28 lotteries that have only negative outcomes. The prediction of CPT in the third column of table 2 is based on the best fitting parameters $\beta = 0.88, \delta = 0.69$ estimated by Tversky and Kahneman (1992).

Lottery	Elicited median CE	CE predicted by CPT $\alpha = 0.88$, $\gamma = 0.61$	CE predicted by StEUT $\alpha = 0.775$, $\sigma = 0.771$
(0,0.9;50,0.1)	9	7.4	7.3
(0,0.5;50,0.5)	21	18.7	20.4
(0,0.1;50,0.9)	37	34.0	35.9
(0,0.95;100,0.05)	14	10.0	8.9
(0,0.75;100,0.25)	25	24.6	27.6
(0,0.5;100,0.5)	36	37.4	40.9
(0,0.25;100,0.75)	52	52.6	55.3
(0,0.05;100,0.95)	78	76.9	80.6
(0,0.99;200,0.01)	10	7.4	5.9
(0,0.9;200,0.1)	20	29.6	29.3
(0,0.5;200,0.5)	76	74.8	81.8
(0,0.1;200,0.9)	131	135.9	143.7
(0,0.01;200,0.99)	188	180.0	183.4
(0,0.99;400,0.01)	12	14.9	11.8
(0,0.01;400,0.99)	377	360.1	366.8
(50,0.9;100,0.1)	59	59.0	60.6
(50,0.5;100,0.5)	71	70.5	74.0
(50,0.1;100,0.9)	83	85.2	88.1
(50,0.95;150,0.05)	64	62.4	63.7
(50,0.75;150,0.25)	72.5	77.7	84.0
(50,0.5;150,0.5)	86	90.5	97.0
(50,0.25;150,0.75)	102	105.3	110.5
(50,0.05;150,0.95)	128	128.4	133.1
(100,0.95;200,0.05)	118	112.7	114.3
(100,0.75;200,0.25)	130	128.2	135.0
(100,0.5;200,0.5)	141	141.1	148.1
(100,0.25;200,0.75)	162	155.8	161.4
(100,0.05;200,0.95)	178	178.7	183.6
Weighted sum of squared errors	0	0.5092	0.6672

Table 1 Tversky and Kahneman (1992) dataset: actual and predicted certainty equivalents for lotteries that have only positive outcomes

Lottery	Elicited median CE	CE predicted by CPT $\beta = 0.88$, $\delta = 0.69$	CE predicted by StEUT $\beta = 0.762$, $\sigma = 0.608$
(-50,0.1;0,0.9)	-8	-6.7	-5.6
(-50,0.5;0,0.5)	-21	-20.4	-20.1
(-50,0.9;0,0.1)	-39	-37.4	-38
(-100,0.05;0,0.95)	-8	-8.3	-6.6
(-100,0.25;0,0.75)	-23.5	-24.8	-23.2
(-100,0.5;0,0.5)	-42	-40.8	-40.3
(-100,0.75;0,0.25)	-63	-58.8	-59.3
(-100,0.95;0,0.05)	-84	-83.1	-83.8
(-200,0.01;0,0.99)	-3	-5.1	-4.1
(-200,0.1;0,0.9)	-23	-26.7	-22.4
(-200,0.50,0.5)	-89	-81.5	-80.5
(-200,0.9;0,0.1)	-155	-149.7	-152.1
(-200,0.99;0,0.01)	-190	-187.6	-186.5
(-400,0.01;0,0.99)	-14	-10.2	-8.3
(-400,0.99;0,0.01)	-380	-375.1	-372.9
(-100,0.1;-50,0.9)	-59	-58.2	-58.8
(-100,0.5;-50,0.5)	-71	-72.2	-74
(-100,0.9;-50,0.1)	-85	-88.4	-90
(-150,0.05;-50,0.95)	-60	-60.4	-61.1
(-150,0.25;-50,0.75)	-71	-78	-80
(-150,0.5;-50,0.5)	-92	-93.8	-96.8
(-150,0.75;-50,0.25)	-113	-111.2	-114.4
(-150,0.95;-50,0.05)	-132	-134.2	-136.1
(-200,0.05;-100,0.95)	-112	-110.7	-111.7
(-200,0.25;-100,0.75)	-121	-128.5	-131
(-200,0.5;-100,0.5)	-142	-144.4	-148
(-200,0.75;-100,0.25)	-158	-161.7	-165.4
(-200,0.95;-100,0.05)	-179	-184.5	-186.5
Weighted sum of squared errors	0	0.6601	0.4889

Table 2 Tversky and Kahneman (1992) dataset: actual and predicted certainty equivalents for lotteries that have only negative outcomes

The fourth column of tables 1 and 2 presents the certainty equivalents predicted by StEUT in equation (4). For estimation purposes, a subjective utility function is defined over changes in wealth exactly as the value function is defined in CPT, e.g. equation (10).

$$u(x)=\begin{cases} x^{\alpha}, & x \geq 0 \\ -\lambda(-x)^{\beta}, & x<0 \end{cases} \quad (10)$$

where α, β and λ are constant. Coefficients α and β capture the curvature of utility function correspondingly for positive and negative outcomes. Coefficient λ captures the loss aversion (e.g. Kahneman and Tversky, 1979; Benartzi and Thaler, 1995).

According to StEUT, the standard deviation of random errors is higher for lotteries with a wider range of possible outcomes, and it converges to zero for lotteries converging to a degenerate lottery. Although many functional forms satisfy these two properties, a parsimonious specification (11) turned out to fit the data most successfully.

$$\sigma_L=\sigma \cdot \left(u(x_n)-u(x_1)\right)\sqrt{\prod_{i=1}^{n}(1-p_i)} \quad (11)$$

where σ is constant across all lotteries. Coefficient σ captures the standard deviation of random errors that is not lottery-specific. For example, in the experiments with hypothetical incentives, σ is expected to be higher than in the experiments with real incentives because real incentives tend to reduce the number of errors (e.g. Smith and Walker, 1993; Harless and Camerer, 1994, p.1265). Subsection 4.11 below demonstrates that actual experimental data support this intuition.

In this section, StEUT is contested primarily against CPT. CPT allows its coefficient capturing the non-linear weighting of probabilities to be different for gains and losses. The coefficient σ from equation (11) is also allowed to be different for lotteries that have only positive outcomes vs. lotteries that have only negative outcomes. With this extra freedom, the parametric versions of StEUT and CPT have exactly the same number of parameters.

The certainty equivalents predicted by StEUT in the fourth column of tables 1 and 2 are calculated through equation (4) using the functional forms (10) and (11). Parameters α, β and σ are estimated to minimize the weighted sum of squared errors $WSSE = \sum_{i=1}^{28}\left(CE_i^{StEUT}/CE_i - 1\right)^2$, where CE_i^{StEUT} is the certainty equivalent predicted by StEUT and CE_i is the certainty equivalent elicited in the experiment for lottery $i \in [1,28]$. Non-linear optimization was implemented in the *Matlab* 6.5 package. The program files are available from the author on request. The best fitting parameters $\alpha = 0.775$ and $\beta = 0.762$ of StEUT utility function are slightly lower than the corresponding parameters of CPT. This indicates a more concave utility function for positive outcomes and a more convex utility function for negative outcomes. The estimated standard deviation of random errors σ is lower for negative outcomes than for positive outcomes. This can be interpreted that subjects are more diligent (less vulnerable to error) when making decisions involving losses.

Tables 1 and 2 demonstrate that both the predictions of CPT and StEUT fit actual experimental data from Tversky and Kahneman (1992) extremely well. CPT fits better (WSSE= 0.5092) than does StEUT (WSSE=0.6672) for lotteries that have only positive outcomes. StEUT fits better (WSSE=0.4889) than does CPT (WSSE=0.6601) for lotteries that have only negative outcomes. For comparison, when the certainty equivalents are calculated through the deterministic EUT, WSSE is 3.788 for lotteries that have only positive outcomes, and WSSE is 3.1958 for lotteries that have only negative outcomes.[2] To summarize, StEUT explains successfully the fourfold pattern of risk attitudes documented in Tversky and Kahneman (1992) and its prediction is at least as good as that of CPT and it is significantly better than that of EUT.

[2] The prediction of the deterministic EUT is calculated using the same utility function as in StEUT, i.e. $\alpha = 0.775$ and $\beta = 0.762$. In fact, the deterministic EUT is just a restricted version of StEUT when $\sigma = 0$.

4.2. Gongalez and Wu (1999)

Gonzalez and Wu (1999) elicited the certainty equivalents for 165 lotteries and found evidence consistent with the non-linear probability weighting. The first number in every cell of table 3 presents a median certainty equivalent elicited for a corresponding lottery. The second number in every cell of table 3 presents the certainty equivalent predicted by CPT with best fitting parameters ($\alpha = 0.49, \gamma = 0.44, \delta = 0.77$) estimated by Gonzalez and Wu (1999).

The third number in every cell of table 3 (presented in the second line) is the certainty equivalent predicted by StEUT. The parametric form of StEUT and the estimation technique are the same as described in section 4.1. The best fitting parameters of StEUT are $\alpha = 0.442$ and $\sigma = 1.403$. The power coefficient of utility function α is slightly lower than the corresponding coefficient of CPT estimated by Gonzalez and Wu (1999). The coefficient σ is almost twice as high for the dataset of Gonzalez and Wu (1999) as for the dataset of Tversky and Kahneman (1992). It is possibly due to a high incidence of weak monotonicity violations (21% of pairwise comparisons) in the data reported by Gonzalez and Wu (1999).

Overall StEUT fits the data from Gonzalez and Wu (1999) better (WSSE is 15.4721) than does CPT (WSSE is 17.4612). The certainty equivalents predicted by CPT are systematically below the actual experimental data for lotteries whose expected utility is close to the utility of the lowest possible outcome, e.g. columns 2-5 of table 3. For these lotteries, StEUT makes a slightly higher prediction and hence it obtains a better result. However, there is clearly room for further improvement because even the prediction of StEUT is quite below the actual certainty equivalents in the upper left region of table 3. For lotteries whose expected utility is close to the utility of the highest possible outcome (e.g. columns 10-12 of table 3), both StEUT and CPT make similar predictions. Thus, “noisy expected utility” is at least as successful as the non-linear probability weighting in explaining the data from Gonzalez and Wu (1999).

ery omes	Probability attached to the higher outcome										
	0.01	**0.05**	**0.1**	**0.25**	**0.4**	**0.5**	**0.6**	**0.75**	**0.9**	**0.95**	**0**
nd 0	4 0.2 0.2	4 0.7 1.2	8 1.2 2.4	9 2.5 4.1	10 3.7 4.8	9.5 4.6 5.2	12 5.6 5.6	11.5 7.5 6.5	14.5 11.0 9.3	13 13.4 12.6	19 1
nd 0	6 0.4 0.4	7 1.4 2.4	8 2.4 4.7	12.5 4.9 8.1	10 7.4 9.6	14 9.1 10.4	12.5 11.1 11.2	19.5 15.1 13.0	22.5 22.0 18.7	27.5 26.9 25.2	40 3
nd 0	5 0.6 0.6	10 2.1 3.6	8.5 3.6 7.1	14 7.4 12.2	17 11.1 14.5	16 13.7 15.6	18 16.7 16.8	23 22.6 19.5	36 33.1 28.0	31.5 40.3 37.7	48. 5
and 0	10 0.8 0.7	10 2.8 4.8	15 4.8 9.5	21 9.9 16.3	19 14.8 19.3	23 18.3 20.8	35 22.3 22.4	31 30.1 26.0	63 44.1 37.3	58 53.8 50.3	84. 7
and 0	10 1.2 1.1	10 4.2 7.2	18 7.2 14.2	25 14.8 24.4	34 22.2 28.9	25 27.4 31.2	49 33.4 33.6	70 45.2 38.9	41.5 66.1 56.0	106 80.6 75.5	118 1
and 0	6 1.6 1.5	9 5.6 9.6	21 9.7 18.9	26.5 19.8 32.6	34.5 29.5 38.6	34 36.6 41.6	56 44.6 44.8	48 60.2 51.9	80 88.2 74.6	102 107.5 100.6	158 1
and 0	18 3.1 3.0	24 11.3 19.2	24 19.3 37.8	54 39.6 65.2	64 59.1 77.2	58 73.2 83.2	58 89.2 89.6	115 120.4 103.8	208.5 176.3 149.2	249 215.0 201.2	277 3
and 0	9.5 6.2 6.0	42 22.6 38.4	36 38.6 75.7	90 79.2 130.3	91 118.2 154.3	89.5 146.3 166.5	207 178.3 179.1	197.5 240.8 207.6	404 352.6 298.4	448.5 430.0 402.5	519 6
nd 25	28 26.9 27.4	29.5 28.7 30.6	30.5 29.9 32.7	32 32.1 35.0	34.5 33.8 35.9	34.5 34.8 36.3	37.5 35.9 36.7	36.5 37.8 37.6	38 40.8 40.1	41.5 42.6 42.5	41 4
nd 50	56.5 52.1 52.6	58 54.0 56.0	56 55.2 58.2	59.5 57.5 60.5	62 59.2 61.4	63 60.2 61.8	64 61.3 62.2	64.5 63.2 63.1	64.5 66.2 65.5	65 67.9 67.9	68. 7
nd 50	58 53.9 54.8	59 57.4 61.2	59 59.8 65.5	62.5 64.2 70.1	66.5 67.5 71.8	70 69.6 72.6	78 71.8 73.4	82.5 75.6 75.2	80 81.5 80.2	78.5 85.2 85.1	89 9
nd 50	57 57.0 58.5	58.5 63.5 70.4	71 67.9 78.5	79 76.2 87.5	89 82.7 90.9	84 86.8 92.5	92 91.1 94.2	99 98.8 97.7	121 110.9 107.8	106 118.5 118.0	117 1
nd 100	114 104.2 105.2	110 108.0 112.0	116.5 110.4 116.4	116 115.0 121.0	121 118.4 122.8	125 120.5 123.6	125.5 122.7 124.4	131 126.5 126.2	133.5 132.3 131.1	130 135.9 135.8	142.5 14
nd 100	111.5 107.8 109.6	115 114.9 122.5	117 119.6 130.9	123 128.4 140.1	131 135.0 143.6	135 139.2 145.2	144 143.6 146.9	146 151.2 150.4	149 163.1 160.3	171 170.4 170.1	158 1
nd 150	156 154.3 155.3	165 158.2 162.3	160 160.7 166.7	166 165.3 171.4	171.5 168.7 173.2	170.5 170.9 174.0	176 173.0 174.8	177 176.9 176.6	187.5 182.7 181.4	179.5 186.2 186.1	190 1

e 3 Gonzalez and Wu (1999) dataset: median certainty equivalents of lotteries as elicited in the experiment (first numb
inty equivalents predicted by CPT (second number) and certainty equivalents predicted by StEUT (third number)

4.3. Wu and Gonzalez (1996)

Table 4 presents 40 decision problems from Wu and Gonzalez (1996). Every decision problem is a binary choice between a safer lottery *S* (presented in the first column of table 4) and a riskier lottery *R* (presented in the second column of table 4). 40 decision problems are divided into 5 blocks of 8 problems. In every block, all 8 problems are constructed by shifting the same probability mass from the medium to the lowest outcome to test for the generalized common consequence effect. The third column of table 4 presents the fraction of subjects *F* who choose a safer lottery *S* in the corresponding problem. The data from Wu and Gonzalez (1996) document a strong common consequence effect. In all five blocks, the fraction *F* first decreases and then increases when the probability mass is shifted from the medium outcome to the lowest outcome.

The fourth column of table 4 presents the probability of choosing *S* over *R* as predicted by CPT. Wu and Gonzalez (1996) calculate this probability using the logistic function proposed by Luce and Suppes (1965): $prob(S \succ R) = 1/\left(1 + e^{V(R)-V(S)}\right)$, where $V(.)$ is the utility of a lottery according to CPT. Wu and Gonzalez (1996) estimate the best fitting parameters of CPT ($\alpha = 0.5022, \gamma = 0.7054$) to minimize the sum of squared errors $SSE = \sum_{i=1}^{40}\left(prob(S_i \succ R_i) - F_i\right)^2$. Minimization of *SSE* is equivalent to maximum likelihood if errors are distributed normally, e.g. Seber and Wild (1989). The fifth column of table 4 presents the probability of choosing *S* over *R* as predicted by StEUT. This probability is calculated through equation (6) using the functional forms (10) and (11). The best fitting parameters of StEUT ($\alpha = 0.172, \sigma = 0.8185$) are estimated to minimize the sum of squared errors *SSE*. Non-linear optimization was implemented in the *Matlab* 6.5 package (program files are available on request). StEUT fits the data from Wu and Gonzalez (1996) slightly better (*SSE*=0.2183) than does CPT (*SSE*=0.2419). Thus, StEUT explains successfully the common consequence effect documented in Wu and Gonzalez (1996).

Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
(0,0.93;200,0.07)	(0,0.95;240,0.05)	0.62	0.5547	0.5525
(0,0.83;200,0.17)	(0,0.85;200,0.1;240,0.05)	0.46	0.5242	0.5149
(0,0.73;200,0.27)	(0,0.75;200,0.2;240,0.05)	0.39	0.5143	0.5046
(0,0.63;200,0.37)	(0,0.65;200,0.3;240,0.05)	0.35	0.5102	0.5002
(0,0.48;200,0.52)	(0,0.5;200,0.45;240,0.05)	0.53	0.5096	0.4987
(0,0.33;200,0.67)	(0,0.35;200,0.6;240,0.05)	0.47	0.5146	0.5032
(0,0.18;200,0.82)	(0,0.2;200,0.75;240,0.05)	0.53	0.5298	0.5216
(0,0.03;200,0.97)	(0,0.05;200,0.9;240,0.05)	0.61	0.6003	0.6291
(0,0.9;50,0.1)	(0,0.95;100,0.05)	0.58	0.5254	0.5906
(0,0.8;50,0.2)	(0,0.85;50,0.1;100,0.05)	0.51	0.4938	0.5204
(0,0.7;50,0.3)	(0,0.75;50,0.2;100,0.05)	0.44	0.4829	0.4986
(0,0.6;50,0.4)	(0,0.65;50,0.3;100,0.05)	0.43	0.4784	0.4885
(0,0.45;50,0.55)	(0,0.5;50,0.45;100,0.05)	0.48	0.4784	0.4836
(0,0.3;50,0.7)	(0,0.35;50,0.6;100,0.05)	0.46	0.4855	0.4889
(0,0.15;50,0.85)	(0,0.2;50,0.75;100,0.05)	0.49	0.5068	0.5135
(50,1)	(0,0.05;50,0.9;100,0.05)	0.78	0.6654	0.7687
(0,0.98;150,0.02)	(0,0.99;300,0.01)	0.48	0.5202	0.5809
(0,0.88;150,0.12)	(0,0.89;150,0.1;300,0.01)	0.44	0.4824	0.4838
(0,0.78;150,0.22)	(0,0.79;150,0.2;300,0.01)	0.31	0.4758	0.4774
(0,0.68;150,0.32)	(0,0.69;150,0.3;300,0.01)	0.3	0.4731	0.4738
(0,0.53;150,0.47)	(0,0.54;150,0.45;300,0.01)	0.42	0.4721	0.4704
(0,0.38;150,0.62)	(0,0.39;150,0.6;300,0.01)	0.4	0.4735	0.4686
(0,0.18;150,0.82)	(0,0.19;150,0.8;300,0.01)	0.46	0.4814	0.4655
(150,1)	(0,0.01;150,0.98;300,0.01)	0.66	0.6057	0.6156
(0,0.95;200,0.05)	(0,0.97;320,0.03)	0.6	0.5348	0.5661
(0,0.85;200,0.15)	(0,0.87;200,0.1;320,0.03)	0.39	0.491	0.5048
(0,0.75;200,0.25)	(0,0.77;200,0.2;320,0.03)	0.46	0.479	0.4931
(0,0.65;200,0.35)	(0,0.67;200,0.3;320,0.03)	0.4	0.474	0.4875
(0,0.5;200,0.5)	(0,0.52;200,0.45;320,0.03)	0.49	0.4727	0.484
(0,0.3;200,0.7)	(0,0.32;200,0.65;320,0.03)	0.53	0.4799	0.4863
(0,0.1;200,0.9)	(0,0.12;200,0.85;320,0.03)	0.54	0.5133	0.5057
(200,1)	(0,0.02;200,0.95;320,0.03)	0.7	0.6976	0.7633
(0,0.95;100,0.05)	(0,0.99;500,0.01)	0.69	0.5609	0.6848
(0,0.85;100,0.15)	(0,0.89;100,0.1;500,0.01)	0.58	0.4775	0.4821
(0,0.75;100,0.25)	(0,0.79;100,0.2;500,0.01)	0.59	0.4587	0.458
(0,0.65;100,0.35)	(0,0.69;100,0.3;500,0.01)	0.43	0.4512	0.4469
(0,0.5;100,0.5)	(0,0.54;100,0.45;500,0.01)	0.56	0.4487	0.4396
(0,0.3;100,0.7)	(0,0.34;100,0.65;500,0.01)	0.52	0.4579	0.4417
(0,0.15;100,0.85)	(0,0.19;100,0.8;500,0.01)	0.53	0.4829	0.4522
(100,1)	(0,0.04;100,0.95;500,0.01)	0.59	0.6845	0.6014
Sum of squared errors		**0**	**0.2419**	**0.2183**

Table 4 Wu and Gonzalez (1996) dataset: the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.5, \gamma = 0.71$) and StEUT ($\alpha = 0.17, \sigma = 0.82$).

4.4. Camerer and Ho (1994)

Camerer and Ho (1994) report experimental results on 12 binary choice problems constructed to test the betweenness axiom. Every decision problem is a binary choice between a safer lottery *S* (presented in the second column of table 5) and a riskier lottery *R* (presented in the third column of table 5). The fourth column of table 5 shows the fraction of subjects who choose a safer lottery *S* in the corresponding decision problem. The fifth column of table 5 presents the probability of choosing *S* over *R* as predicted by CPT. Camerer and Ho (1994) use the logistic function (e.g. Luce and Suppes, 1965, p.335) to calculate $prob(S \succ R)$ as described in section 4.3. The best fitting parameters of CPT ($\alpha = 0.6008$, $\gamma = 1.0306$), not reported in Camerer and Ho (1994), are estimated to minimize *SSE*, which is equivalent to maximum likelihood estimation of Camerer and Ho (1994) if errors are distributed normally. The sixth column of table 5 presents the probability of choosing *S* over *R* as predicted by StEUT. The parametric form of StEUT and the estimation technique are the same as described in section 4.3.

The best fitting parameters of StEUT that minimize *SEE* are $\alpha = 0.5215, \sigma = 0.1242$.

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(0,0.3;80,0.4;200,0.3)	(0,0.5;200,0.5)	0.7093	0.6814	0.6999
2	(0,0.3;80,0.4;200,0.3)	(0,0.4;80,0.2;200,0.4)	0.6512	0.5937	0.6
3	(0,0.4;80,0.2;200,0.4)	(0,0.5;200,0.5)	0.5581	0.5941	0.6039
4	(0,0.4;80,0.6)	(0,0.6;80,0.2;200,0.2)	0.7711	0.7089	0.7438
5	(0,0.4;80,0.6)	(0,0.5;80,0.4;200,0.1)	0.4458	0.6156	0.6259
6	(0,0.5;80,0.4;200,0.1)	(0,0.6;80,0.2;200,0.2)	0.6867	0.6033	0.6051
7	(0,0.5;80,0.4;200,0.1)	(0,0.7;200,0.3)	0.7654	0.692	0.7115
8	(0,0.5;80,0.4;200,0.1)	(0,0.6;80,0.2;200,0.2)	0.6543	0.6033	0.6051
9	(0,0.6;80,0.2;200,0.2)	(0,0.7;200,0.3)	0.5062	0.5964	0.6115
10	(0,0.66;120,0.34)	(0,0.83;200,0.17)	0.9151	0.8822	0.9183
11	(0,0.66;120,0.34)	(0,0.67;120,0.32;200,0.01)	0.1792	0.5315	0.5287
12	(0,0.67;120,0.32;200,0.01)	(0,0.83;200,0.17)	0.9245	0.8685	0.8721
Sum of squared errors			**0**	**0.1895**	**0.186**

Table 5 Camerer and Ho (1994) dataset: the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.6, \gamma = 1.03$) and StEUT ($\alpha = 0.52, \sigma = 0.12$).

Both CPT and StEUT explain a revealed actual choice in all decision problems except in problem #11 extremely well. Problem #11 is the replication of a hypothetical choice problem originally reported in Prelec (1990). Interestingly, problem #11 is the only decision problem in which Camerer and Ho (1994) document a strong asymmetric split between quasi-concave and quasi-convex preferences when the modal choice is not consistent with the betweenness axiom. In all other decision problems where Camerer and Ho (1994) find also an asymmetric split between quasi-concave and quasi-convex preferences, i.e. an evidence of betweenness violation, the modal choice is, nevertheless, consistent with the betwenness axiom.

StEUT has only a marginally better fit (*SSE*=0.186) when compared with CPT (*SSE*=0.1895). Notice that the best fitting parameter $\gamma = 1.0306$ of CPT is not conventional and the conventional parameterizations of CPT, e.g. $\gamma < 1$ would have even worse fit. Thus, StEUT explains successfully the violations of betwenness documented in Camerer and Ho (1994). However, StEUT does not explain one specific violation of betweenness (#11), when a revealed modal choice is not consistent with the betweenness axiom. It is interesting that CPT does not explain this particular violation either.

4.5. Bernasconi (1994)

Table 6 presents 20 problems of binary choice from Bernasconi (1994) designed to test the betweenness axiom[3]. The format of table 6 is the same as for table 5. The predictions of CPT and StEUT are based on the parametric forms and the estimation technique already described in section 4.3. The best fitting parameters of CPT are $\alpha = 0.5728, \gamma = 0.4388$. Bernasconi (1994) does not report the fit of CPT to her experimental data. This exercise is done here for comparison vs. StEUT. The best fitting parameters of StEUT are $\alpha = 0.2097, \sigma = 0.2766$.

[3] Bernasconi (1994) also presents 8 decision problems involving two-stage lotteries. Those problems are not considered here because both StEUT and CPT deal with single-stage (reduced form) lotteries.

#	Lottery S	Lottery R	Choice of S	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(10,1)	(0,0.2;16,0.8)	0.94	0.8401	0.8615
2	(10,1)	(0,0.01;10, 0.95;16,0.04)	0.36	0.6714	0.562
3	(0,0.01;10, 0.95;16,0.04)	(0,0.2;16,0.8)	0.88	0.72	0.8069
4	(10,1)	(0,0.19;10, 0.05;16,0.76)	0.9	0.8404	0.8445
5	(0,0.19;10, 0.05;16,0.76)	(0,0.2;16,0.8)	0.8	0.4994	0.5058
6	(0,0.8;10, 0.2)	(0,0.84;16,0.16)	0.54	0.4489	0.553
7	(0,0.8;10, 0.2)	(0,0.81;10, 0.15;16,0.04)	0.06	0.4586	0.5058
8	(0,0.81;10, 0.15;16,0.04)	(0,0.84;16,0.16)	0.72	0.4902	0.5455
9	(0,0.8;10, 0.2)	(0,0.83;10, 0.05;16,0.12)	0.5	0.4501	0.5365
10	(0,0.83;10, 0.05;16,0.12)	(0,0.84;16,0.16)	0.24	0.4988	0.5159
11	(0,0.2;12, 0.8)	(0,0.4;20,0.6)	0.74	0.4737	0.7528
12	(0,0.2;12, 0.8)	(0,0.21;12, 0.76;20,0.03)	0.04	0.4567	0.5027
13	(0,0.21;12, 0.76;20,0.03)	(0,0.4;20,0.6)	0.84	0.5171	0.7336
14	(0,0.2;12, 0.8)	(0,0.39;12, 0.04;20,0.57)	0.76	0.4736	0.7384
15	(0,0.39;12, 0.04;20,0.57)	(0,0.4;20,0.6)	0.1	0.5001	0.5119
16	(0,0.84;12, 0.16)	(0,0.88;20,0.12)	0.36	0.4439	0.5662
17	(0,0.84;12, 0.16)	(0,0.85;12, 0.12;20,0.03)	0.12	0.4538	0.5059
18	(0,0.85;12, 0.12;20,0.03)	(0,0.88;20,0.12)	0.76	0.49	0.5583
19	(0,0.84;12, 0.16)	(0,0.87;12, 0.04;20,0.09)	0.52	0.4446	0.545
20	(0,0.87;12, 0.04;20,0.09)	(0,0.88;20,0.12)	0.26	0.4993	0.5206
Sum of squared errors			**0**	**1.3609**	**1.1452**

Table 6 Bernasconi (1994) dataset: the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.57, \gamma = 0.44$) and StEUT ($\alpha = 0.21, \sigma = 0.27$).

StEUT fits to the data in Bernasconi (1994) better (*SSE*=1.1452) than does CPT (*SSE*=1.3609). Notice that problems #1 and #6 as well as #11 and #16 constitute together two pairs of common ratio problems. StEUT predicts correctly the common ratio effect: the fraction of subjects choosing *S* over *R* is much lower in problems #6 and #16 than in problems #1 and #11 (the prediction of CPT is less accurate). However, StEUT fails to explain the choice pattern observed in problems #7, #12, #15 and #17 (the prediction of CPT is only slightly more accurate). These are the problems in which Bernasconi (1994) finds a strong asymmetric split between quasi-concave and quasi-convex preferences, i.e. an evidence of betwenness violation, when the modal choice is not consistent with the betweenness axiom. We already observed a similar predictive failure of both StEUT and CPT in problem #11 from table 5.

4.6. Camerer (1992)

Table 7 (8, 9) presents 9 problems of binary choice between lotteries with large positive (small positive, negative) outcomes from Camerer (1992). In every table all lotteries have the same range of possible outcomes (lotteries are located inside the probability triangle). The format of tables 7, 8 and 9 is the same as for table 5. The predictions of CPT and StEUT are based on the parametric forms and the estimation technique already described in section 4.3.

Camerer (1992) does not find any significant evidence of the common consequence and common ratio effects. Thus, deterministic EUT explains the data in Camerer (1992) quite well. In tables 7 and 8 the best fitting parametric form of CPT has a coefficient γ close to unity, which signifies a minimum departure from linear probability weighting. Similarly, in tables 7 and 8 the best fitting parametric form of StEUT has a coefficient σ close to zero, which denotes a minimum departure from the deterministic EUT. Departures from the deterministic EUT are more apparent only in choices involving lotteries with negative outcomes, e.g. table 9. StEUT fits the data in Camerer (1992) better than does CPT, when lotteries involve small positive or negative outcomes. CPT fits better than does StEUT, when lotteries have large positive outcomes.

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(0,0.65;10,0.25;25,0.1) [4]	(0,0.75;10,0.05;25,0.2)	0.54	0.5793	0.6164
2	(0,0.55;10,0.3;25,0.15)	(0,0.65;10,0.1;25,0.25)	0.58	0.5833	0.6073
3	(0,0.35;10,0.4;25,0.25)	(0,0.45;10,0.2;25,0.35)	0.59	0.5982	0.6003
4	(0,0.05;10,0.55;25,0.4)	(0,0.15;10,0.35;25,0.5)	0.68	0.6575	0.6045
5	(0,0.05;10,0.5;25,0.45)	(0,0.15;10,0.35;25,0.5)	0.67	0.6581	0.6048
6	(0,0.15;10,0.3;25,0.55)	(0,0.25;10,0.1;25,0.65)	0.57	0.6282	0.6073
7	(0,0.05;10,0.9;25,0.05)	(0,0.15;10,0.7;25,0.15)	0.68	0.6062	0.6364
8	(0,0.35;10,0.3;25,0.35)	(0,0.45;10,0.1;25,0.45)	0.59	0.6059	0.6012
9	(0,0.45;10,0.5;25,0.05)	(0,0.55;10,0.3;25,0.15)	0.58	0.5515	0.6048
Sum of squared errors			**0**	**0.0122**	**0.0207**

Table 7 Camerer (1992) dataset (lotteries with large positive outcomes): the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.7542, \gamma = 0.9988$) and StEUT ($\alpha = 0.5871, \sigma = 0.0868$).

[4] All outcomes of lotteries in table 7 are in thousand USD.

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(0,0.65;5,0.25;10,0.1)	(0,0.75;5,0.05;10,0.2)	0.52	0.53	0.5418
2	(0,0.55;5,0.3;10,0.15)	(0,0.65;5,0.1;10,0.25)	0.50	0.5298	0.5384
3	(0,0.35;5,0.4;10,0.25)	(0,0.45;5,0.2;10,0.35)	0.54	0.5305	0.5359
4	(0,0.05;5,0.55;10,0.4)	(0,0.15;5,0.35;10,0.5)	0.54	0.5371	0.5374
5	(0,0.05;5,0.5;10,0.45)	(0,0.15;5,0.35;10,0.5)	0.51	0.5371	0.5375
6	(0,0.15;5,0.3;10,0.55)	(0,0.25;5,0.1;10,0.65)	0.53	0.5335	0.5385
7	(0,0.05;5,0.9;10,0.05)	(0,0.15;5,0.7;10,0.15)	0.58	0.5346	0.5492
8	(0,0.35;5,0.3;10,0.35)	(0,0.45;5,0.1;10,0.45)	0.62	0.5309	0.5362
9	(0,0.45;5,0.5;10,0.05)	(0,0.55;5,0.3;10,0.15)	0.51	0.5282	0.5375
Sum of squared errors			**0**	**0.0122**	**0.0115**

Table 8 Camerer (1992) dataset (lotteries with small positive outcomes): the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.4989, \gamma = 0.9396$) and StEUT ($\alpha = 0.9123, \sigma = 0.0914$).

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(-10,0.1;-5,0.25;0,0.65)	(-10,0.2;-5,0.05;0,0.75)	0.39	0.477	0.4105
2	(-10,0.15;-5,0.3;0,0.55)	(-10,0.25;-5,0.1;0,0.65)	0.44	0.4564	0.4093
3	(-10,0.25;-5,0.4;0,0.35)	(-10,0.35;-5,0.2;0,0.45)	0.37	0.4276	0.4107
4	(-10,0.4;-5,0.55;0,0.05)	(-10,0.5;-5,0.35;0,0.15)	0.44	0.42	0.4068
5	(-10,0.45;-5,0.5;0,0.05)	(-10,0.5;-5,0.35;0,0.15)	0.36	0.4247	0.4066
6	(-10,0.55;-5,0.3;0,0.15)	(-10,0.65;-5,0.1;0,0.25)	0.33	0.4399	0.4042
7	(-10,0.05;-5,0.9;0,0.05)	(-10,0.15;-5,0.7;0,0.15)	0.48	0.3944	0.3776
8	(-10,0.35;-5,0.3;0,0.35)	(-10,0.45;-5,0.1;0,0.45)	0.36	0.4379	0.4096
9	(-10,0.05;-5,0.5;0,0.45)	(-10,0.15;-5,0.3;0,0.55)	0.45	0.4306	0.4117
Sum of squared errors			**0**	**0.0416**	**0.0262**

Table 9 Camerer (1992) dataset (lotteries with negative outcomes): the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\beta = 0.5423, \delta = 2.9945$) and StEUT ($\beta = 0.5182, \sigma = 0.2299$).

4.7. Camerer (1989)

Table 10 (11, 12) presents 14 problems of binary choice between lotteries with large positive (small positive, negative) outcomes from Camerer (1989). The problems are designed to test the betweenness axiom, the common consequence effect and the fourfold pattern of risk attitudes. The format of tables 7, 8 and 9 is the same as for table 5. The predictions of CPT and StEUT are based on the parametric forms and the estimation technique described in section 4.3.

Similarly as in the dataset from Camerer (1992), StEUT explains the data reported in Camerer (1989) better than does CPT when lotteries involve small positive or negative outcomes. CPT fits better than StEUT when lotteries have large positive outcomes. However, both CPT and StEUT fail to explain the choice pattern in problems #9 and #12 in table 10, i.e. the frequent choice of a riskier lottery *R* that delivers large positive outcome with a small probability. This can be related to the predictive failure of CPT and StEUT in the upper left region of table 3.

#	**Lottery *S***	**Lottery R**	**Choice of *S***	$prob(S \succ R)$ **predicted by CPT**	$prob(S \succ R)$ **predicted by StEUT**
1	(10,0.2;25,0.8) [5]	(0,0.1;25,0.9)	0.73	0.7371	0.7976
2	(10,0.6;25,0.4)	(0,0.1;10,0.4;25,0.5)	0.75	0.7437	0.7479
3	(10,0.6;25,0.4)	(0,0.2;10,0.2;25,0.6)	0.79	0.8162	0.7882
4	(0,0.1;10,0.4;25,0.5)	(0,0.3;25,0.7)	0.71	0.6739	0.5769
5	(10,1)	(0,0.1;10,0.8;25,0.1)	0.71	0.727	0.6456
6	(10,1)	(0,0.2;10,0.6;25,0.2)	0.8	0.8005	0.718
7	(0,0.3;10,0.4;25,0.3)	(0,0.5;25,0.5)	0.69	0.6162	0.5983
8	(0,0.4;10,0.2;25,0.4)	(0,0.5;25,0.5)	0.46	0.5557	0.5503
9	(0,0.4;10,0.6)	(0,0.5;10,0.4;25,0.1)	0.28	0.5346	0.5305
10	(0,0.4;10,0.6)	(0,0.6;10,0.2;25,0.2)	0.75	0.5831	0.5924
11	(0,0.5;10,0.4;25,0.1)	(0,0.7;25,0.3)	0.68	0.6016	0.618
12	(0,0.8;10,0.2)	(0,0.9;25,0.1)	0.27	0.5595	0.5778
13	(0,0.2;10,0.2;25,0.6)	(0,0.3;25,0.7)	0.53	0.5745	0.5363
14	(0,0.6;10,0.2;25,0.2)	(0,0.7;25,0.3)	0.4	0.5536	0.565
Sum of squared errors			**0**	**0.1996**	**0.2359**

Table 10 Camerer (1989) dataset (lotteries with large positive outcomes): the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.1617, \gamma = 0.6566$**) and StEUT (**$\alpha = 0.3037, \sigma = 0.4816$**).**

[5] All outcomes of lotteries in table 10 are in thousand USD.

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(5,0.2;10,0.8)	(0,0.1;10,0.9)	0.59	0.5481	0.6698
2	(5,0.6;10,0.4)	(0,0.1;5,0.4;10,0.5)	0.65	0.5517	0.5903
3	(5,0.6;10,0.4)	(0,0.2;5,0.2;10,0.6)	0.64	0.5882	0.644
4	(0,0.1;5,0.4;10,0.5)	(0,0.3;10,0.7)	0.63	0.5687	0.5837
5	(5,1)	(0,0.1;5,0.8;10,0.1)	0.48	0.543	0.5831
6	(5,1)	(0,0.2;5,0.6;10,0.2)	0.58	0.5767	0.6321
7	(0,0.3;5,0.4;10,0.3)	(0,0.5;10,0.5)	0.74	0.5585	0.5903
8	(0,0.4;5,0.2;10,0.4)	(0,0.5;10,0.5)	0.47	0.529	0.5458
9	(0,0.4;5,0.6)	(0,0.5;5,0.4;10,0.1)	0.31	0.5202	0.5504
10	(0,0.4;5,0.6)	(0,0.6;5,0.2;10,0.2)	0.74	0.5458	0.6068
11	(0,0.5;5,0.4;10,0.1)	(0,0.7;10,0.3)	0.76	0.554	0.598
12	(0,0.8;5,0.2)	(0,0.9;10,0.1)	0.46	0.5271	0.5527
13	(0,0.2;5,0.2;10,0.6)	(0,0.3;10,0.7)	0.37	0.5319	0.5421
14	(0,0.6;5,0.2;10,0.2)	(0,0.7;10,0.3)	0.45	0.5284	0.5515
Sum of squared errors			**0**	**0.1871**	**0.1639**

Table 11 Camerer (1989) dataset (lotteries with small positive outcomes): the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.5201, \gamma = 0.8896$) and StEUT ($\alpha = 0.683, \sigma = 0.2897$).

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(-10,0.8;-5,0.2)	(-10,0.9;0,0.1)	0.54	0.386	0.3874
2	(-10,0.4;-5,0.6)	(-10,0.5;-5,0.4;0,0.1)	0.48	0.3789	0.4138
3	(-10,0.4;-5,0.6)	(-10,0.6;-5,0.2;0,0.2)	0.21	0.3411	0.3283
4	(-10,0.5;-5,0.4;0,0.1)	(-10,0.7;0,0.3)	0.21	0.4334	0.3499
5	(-5,1)	(-10,0.1;-5,0.8;0,0.1)	0.27	0.3972	0.3698
6	(-5,1)	(-10,0.2;-5,0.6;0,0.2)	0.20	0.3615	0.2985
7	(-10,0.3;-5,0.4;0,0.3)	(-10,0.5;0,0.5)	0.34	0.4613	0.3587
8	(-10,0.4;-5,0.2;0,0.4)	(-10,0.5;0,0.5)	0.51	0.4821	0.4278
9	(-5,0.6;0,0.4)	(-10,0.1;-5,0.4;0,0.5)	0.33	0.5012	0.3912
10	(-5,0.6;0,0.4)	(-10,0.2;-5,0.2;0,0.6)	0.35	0.488	0.3023
11	(-10,0.1;-5,0.4;0,0.5)	(-10,0.3;0,0.7)	0.51	0.4705	0.356
12	(-5,0.2;0,0.8)	(-10,0.1;0,0.9)	0.40	0.4912	0.2985
13	(-10,0.6;-5,0.2;0,0.2)	(-10,0.7;0,0.3)	0.40	0.474	0.4222
14	(-10,0.2;-5,0.2;0,0.6)	(-10,0.3;0,0.7)	0.40	0.4838	0.4252
Sum of squared errors			**0**	**0.217**	**0.1281**

Table 12 Camerer (1989) dataset (lotteries with negative outcomes): the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\beta = 0.3882, \delta = 0.6185$) and StEUT ($\beta = 0.6207, \sigma = 0.2252$).

4.8. Conlisk (1989)

Table 13 presents 5 binary choice problems from Conlisk (1989). Table 13 has the same format as table 5. Conlisk (1989) replicates the Allais paradox in problems #1 and #2. Problems #3 and #4 constitute a similar common consequence problem to the Allais paradox; however, they do not employ a degenerate lottery that delivers one million for certain. Table 13 shows that the incidence of the Allais paradox completely disappears in problems #3 and #4. Finally, problems #1 and #5 constitute a variant of the Allais paradox, when a probability mass is shifted from the medium to the highest (not lowest) outcome. Table 13 shows that the switch in preferences between lotteries *S* and *R* across problems #1 and #5 is comparable to that in problems #1 and #2 (the original Allais paradox).

StEUT with the estimated best fitting parameters $\alpha = 0.5049, \sigma = 1.858$ explains the data in Conlisk (1989) marginally better than does CPT with the estimated best fitting parameters $\alpha = 0.2241, \gamma = 0.6276$. CPT predicts very well the original Allais paradox; however, it also predicts the common consequence effect for problems #3 and #4, which is not found in the data. StEUT makes a less accurate prediction for the original Allais paradox but it predicts no common consequence effect for problems #3 and #4.

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	$(10^6,1)$	$(0,0.01;10^6,0.89;5*10^6,0.1)$	0.5127	0.4989	0.4378
2	$(0,0.89;10^6,0.11)$	$(0,0.9; 5*10^6,0.1)$	0.1441	0.1743	0.25
3	$(0,0.01;10^6,0.89;5*10^6,0.1)$	$(0,0.02;10^6,0.78;5*10^6,0.2)$	0.4651	0.5248	0.4924
4	$(0,0.71;10^6,0.19;5*10^6,0.1)$	$(0,0.72;10^6,0.08;5*10^6,0.2)$	0.4651	0.3502	0.495
5	$(0,0.01;10^6,0.11;5*10^6,0.88)$	$(0,0.02; 5*10^6,0.98)$	0.25	0.292	0.2823
Sum of squared errors			**0**	**0.0196**	**0.0195**

Table 13 Conlisk (1989) dataset: the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.2241, \gamma = 0.6276$) and StEUT ($\alpha = 0.5049, \sigma = 1.858$).

4.9. Loomes and Sugden (1998)

Table 14 presents 45 binary choice problems from Loomes and Sugden (1998) designed to test the common consequence effect (e.g. ## 1-2-4-5-8), the common ratio effect (e.g. ## 1-3-6-8) and the dominance relation (## 41-45). Every problem was presented to one group of subjects twice. The fourth column of table 14 shows average choice of a safer lottery *S* across all subjects and two trials. Table 15 follows the format of table 14 and presents another 45 binary choice problems from Loomes and Sugden (1998) that were given to another group of subjects.

In the experiment of Loomes and Sugden (1998) the subjects were randomly assigned to one of the two groups. Therefore, a representative agent may be expected to be the same for both groups. The functional forms of CPT and StEUT are restricted to have the same parameters for 45 problems from table 14 and 45 problems from table 15. For this aggregate dataset the best fitting parameters of StEUT are $\alpha = 0.3514, \sigma = 0.1383$ and the best fitting parameters of CPT are $\alpha = 0.4704, \gamma = 0.7537$. StEUT fits the aggregate data (tables 14 and 15 combined together) much better (*SSE*=2.2116) than does CPT (*SSE*=5.6009). CPT predicts correctly the direction of the common consequence and common ratio effects but the effect predicted by CPT is typically weaker than the actual choice patterns.

In problems ## 41-45 from tables 14 and 15 lottery *S* always dominates lottery *R*. Actual experimental data reveal that subjects respect this dominance relation very well. Lottery *S* is chosen in more than 95% of all cases in problems ## 41-44 and it is always chosen in problem #45. CPT embedded in the logistic stochastic utility predicts that *S* is chosen at most in 57% of all cases (e.g. column 5 of tables 14 and 15). Probability $prob(S \succ R)$ predicted by StEUT is closer to the observed choice pattern. Nevertheless, StEUT predicts many more violations of dominance than are actually observed, especially in problems #41 and #45.

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(10,0.25;30,0.75)	(0,0.15;30,0.85)	0.837	0.6446	0.9079
2	(0,0.15;10,0.25;30,0.6)	(0,0.3;30,0.7)	0.8152	0.5613	0.771
3	(10,0.5;30,0.5)	(0,0.3;30,0.7)	0.8804	0.7094	0.9815
4	(10,0.5;30,0.5)	(0,0.15;10,0.25;30,0.6)	0.8587	0.6561	0.8308
5	(10,1)	(0,0.15;10,0.75;30,0.1)	0.6957	0.6103	0.8763
6	(10,1)	(0,0.6;30,0.4)	0.9348	0.7285	1
7	(0,0.15;10,0.75;30,0.1)	(0,0.6;30,0.4)	0.9022	0.6314	0.9896
8	(0,0.75;10,0.25)	(0,0.9;30,0.1)	0.75	0.5125	0.8849
9	(10,0.2;30,0.8)	(0,0.1;30,0.9)	0.8152	0.5977	0.7938
10	(0,0.1;10,0.8;30,0.1)	(0,0.5;30,0.5)	0.837	0.5916	0.9476
11	(10,1)	(0,0.5;30,0.5)	0.7717	0.6564	0.9954
12	(10,1)	(0,0.1;10,0.8;30,0.1)	0.4783	0.5688	0.7407
13	(0,0.5;10,0.4;30,0.1)	(0,0.7;30,0.3)	0.8478	0.5158	0.7738
14	(0,0.4;10,0.6)	(0,0.7;30,0.3)	0.8152	0.4824	0.916
15	(0,0.4;10,0.6)	(0,0.5;10,0.4;30,0.1)	0.3587	0.4667	0.6634
16	(0,0.8;10,0.2)	(0,0.9;30,0.1)	0.5217	0.4834	0.7379
17	(10,0.25;30,0.75)	(0,0.1;30,0.9)	0.6087	0.5757	0.6743
18	(0,0.1;10,0.75;30,0.15)	(0,0.4;30,0.6)	0.7717	0.5309	0.7453
19	(10,1)	(0,0.4;30,0.6)	0.75	0.5746	0.8807
20	(10,1)	(0,0.1;10,0.75;30,0.15)	0.413	0.5441	0.6289
21	(0,0.6;10,0.25;30,0.15)	(0,0.7;30,0.3)	0.6413	0.4906	0.5839
22	(0,0.5;10,0.5)	(0,0.7;30,0.3)	0.7174	0.4315	0.6937
23	(0,0.5;10,0.5)	(0,0.6;10,0.25;30,0.15)	0.3913	0.4407	0.5938
24	(0,0.75;10,0.25)	(0,0.85;30,0.15)	0.4348	0.4514	0.6221
25	(10,0.3;30,0.7)	(0,0.1;30,0.9)	0.5217	0.5552	0.5384
26	(0,0.2;10,0.6;30,0.2)	(0,0.4;30,0.6)	0.7609	0.4733	0.5324
27	(0,0.1;10,0.9)	(0,0.4;30,0.6)	0.6413	0.4219	0.5615
28	(0,0.1;10,0.9)	(0,0.2;10,0.6;30,0.2)	0.2609	0.4481	0.5181
29	(0,0.5;10,0.3;30,0.2)	(0,0.6;30,0.4)	0.6196	0.4756	0.5158
30	(0,0.4;10,0.6)	(0,0.6;30,0.4)	0.5326	0.3966	0.5385
31	(0,0.4;10,0.6)	(0,0.5;10,0.3;30,0.2)	0.2935	0.4203	0.5183
32	(0,0.7;10,0.3)	(0,0.8;30,0.2)	0.3696	0.4253	0.5224
33	(10,0.4;30,0.6)	(0,0.1;30,0.9)	0.4565	0.5174	0.2765
34	(0,0.1;10,0.6;30,0.3)	(0,0.25;30,0.75)	0.5	0.4481	0.3229
35	(10,1)	(0,0.25;30,0.75)	0.337	0.4316	0.1203
36	(10,1)	(0,0.1;10,0.6;30,0.3)	0.3043	0.4833	0.3425
37	(0,0.4;10,0.6)	(0,0.5;10,0.2;30,0.3)	0.25	0.3828	0.3727
38	(0,0.4;10,0.6)	(0,0.55;30,0.45)	0.4565	0.3568	0.3055
39	(0,0.5;10,0.2;30,0.3)	(0,0.55;30,0.45)	0.5	0.4721	0.4444
40	(0,0.6;10,0.4)	(0,0.7;30,0.3)	0.2935	0.3831	0.36
41	(10,0.25;30,0.75)	(10,0.3;30,0.7)	0.9891	0.5208	0.5602
42	(0,0.55;10,0.2;30,0.25)	(0,0.65;10,0.15;30,0.2)	0.9891	0.5702	0.8031
43	(0,0.8;30,0.2)	(0,0.85;30,0.15)	0.9891	0.5536	0.7499
44	(0,0.1;10,0.75;30,0.15)	(0,0.15;10,0.75;30,0.1)	0.9783	0.5675	0.7207
45	(0,0.7;10,0.3)	(0,0.75;10,0.25)	1	0.5273	0.6747

Table 14 Loomes and Sugden (1998) dataset: the fraction of subjects choosing *S* over *R* (group 1) and the prediction of CPT ($\alpha = 0.4633, \gamma = 0.7884$) and StEUT ($\alpha = 0.3514, \sigma = 0.1383$).

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by CPT	$prob(S \succ R)$ predicted by StEUT
1	(10,0.25;20,0.75)	(0,0.15;20,0.85)	0.9239	0.6633	0.9712
2	(0,0.15;10,0.25;20,0.6)	(0,0.3;20,0.7)	0.9674	0.5774	0.8455
3	(10,0.5;20,0.5)	(0,0.3;20,0.7)	0.9783	0.7348	0.9984
4	(10,0.5;20,0.5)	(0,0.15;10,0.25;20,0.6)	0.9565	0.6697	0.9123
5	(10,1)	(0,0.15;10,0.75;20,0.1)	0.8152	0.6444	0.9437
6	(10,1)	(0,0.6;20,0.4)	0.9783	0.7929	1
7	(0,0.15;10,0.75;20,0.1)	(0,0.6;20,0.4)	0.9891	0.6788	0.9992
8	(0,0.75;10,0.25)	(0,0.9;20,0.1)	0.9565	0.5489	0.9369
9	(10,0.2;20,0.8)	(0,0.1;20,0.9)	0.9022	0.6203	0.9132
10	(0,0.1;10,0.8;20,0.1)	(0,0.5;20,0.5)	0.9891	0.6548	0.9948
11	(10,1)	(0,0.5;20,0.5)	0.9239	0.7433	1
12	(10,1)	(0,0.1;10,0.8;20,0.1)	0.6957	0.6042	0.846
13	(0,0.5;10,0.4;20,0.1)	(0,0.7;20,0.3)	1	0.5527	0.8822
14	(0,0.4;10,0.6)	(0,0.7;20,0.3)	0.9891	0.5558	0.9804
15	(0,0.4;10,0.6)	(0,0.5;10,0.4;20,0.1)	0.75	0.5031	0.7376
16	(0,0.8;10,0.2)	(0,0.9;20,0.1)	0.837	0.5199	0.8269
17	(10,0.25;20,0.75)	(0,0.1;20,0.9)	0.7174	0.608	0.862
18	(0,0.1;10,0.75;20,0.15)	(0,0.4;20,0.6)	0.8913	0.6014	0.9357
19	(10,1)	(0,0.4;20,0.6)	0.9239	0.6851	0.9967
20	(10,1)	(0,0.1;10,0.75;20,0.15)	0.5761	0.5905	0.7762
21	(0,0.6;10,0.25;20,0.15)	(0,0.7;20,0.3)	0.8587	0.5169	0.6872
22	(0,0.5;10,0.5)	(0,0.7;20,0.3)	0.8804	0.5047	0.8651
23	(0,0.5;10,0.5)	(0,0.6;10,0.25;20,0.15)	0.6522	0.4878	0.6986
24	(0,0.75;10,0.25)	(0,0.85;20,0.15)	0.75	0.4985	0.75
25	(10,0.3;20,0.7)	(0,0.1;20,0.9)	0.7065	0.5967	0.7961
26	(0,0.2;10,0.6;20,0.2)	(0,0.4;20,0.6)	0.8696	0.5357	0.7651
27	(0,0.1;10,0.9)	(0,0.4;20,0.6)	0.8261	0.5404	0.9194
28	(0,0.1;10,0.9)	(0,0.2;10,0.6;20,0.2)	0.5217	0.5048	0.6736
29	(0,0.5;10,0.3;20,0.2)	(0,0.6;20,0.4)	0.7609	0.5077	0.6379
30	(0,0.4;10,0.6)	(0,0.6;20,0.4)	0.8696	0.4841	0.7933
31	(0,0.4;10,0.6)	(0,0.5;10,0.3;20,0.2)	0.5761	0.4764	0.6515
32	(0,0.7;10,0.3)	(0,0.8;20,0.2)	0.5978	0.4815	0.682
33	(10,0.5;20,0.5)	(0,0.1;20,0.9)	0.5543	0.5559	0.431
34	(0,0.1;10,0.5;20,0.4)	(0,0.2;20,0.8)	0.5326	0.4934	0.4637
35	(10,1)	(0,0.2;20,0.8)	0.5435	0.53	0.3849
36	(10,1)	(0,0.1;10,0.5;20,0.4)	0.413	0.5365	0.4558
37	(0,0.25;10,0.75)	(0,0.35;10,0.25;20,0.4)	0.4674	0.4413	0.4637
38	(0,0.25;10,0.75)	(0,0.4;20,0.6)	0.6413	0.4285	0.4417
39	(0,0.35;10,0.25;20,0.4)	(0,0.4;20,0.6)	0.4348	0.487	0.4841
40	(0,0.5;10,0.5)	(0,0.6;20,0.4)	0.4457	0.4332	0.463
41	(10,0.25;20,0.75)	(10,0.3;20,0.7)	0.9457	0.5118	0.4733
42	(0,0.55;10,0.2;20,0.25)	(0,0.65;10,0.15;20,0.2)	0.9891	0.5617	0.8174
43	(0,0.8;20,0.2)	(0,0.85;20,0.15)	0.9891	0.5442	0.7499
44	(0,0.1;10,0.75;20,0.15)	(0,0.15;10,0.75;20,0.1)	1	0.5569	0.7207
45	(0,0.7;10,0.3)	(0,0.75;10,0.25)	1	0.5273	0.6906

Table 15 Loomes and Sugden (1998) dataset: the fraction of subjects choosing *S* over *R* (group 2) and the prediction of CPT ($\alpha = 0.4633, \gamma = 0.7884$) and StEUT ($\alpha = 0.3514, \sigma = 0.1383$).

4.10. Hey and Orme (1994)

In the experiment of Hey and Orme (1994) 80 subjects made 100 choice decisions on two separate trials. Every decision was a choice between two lotteries with a possibility to declare indifference. Using the same functional forms and non-linear optimization technique as already described in section 4.3, the parameters of CPT and StEUT are found that fit best to each of 80 individual choice patterns (aggregated across two trials). The incidences of indifference are treated as 50%-50% chance to choose either lottery. For 34 out of 80 subjects the best fitting power coefficient α of the value function of CPT turns out to be zero which indicates the violation of outcome monotonicity. These subjects are excluded from further analysis because no sensible comparison between StEUT and CPT is possible in this case.

The data for the remaining 46 subjects are presented in table 16. For convenience, the numbering of subjects in the first column of table 16 corresponds to the numbering in the original Hey and Orme (1994) study. Columns 2-3 (5-6) of table 16 present correspondingly the best fitting parameters of CPT (StEUT) that are estimated separately for every subject to minimize his or her sum of squared errors across all 100 choice problems. The median estimates of CPT parameters are $\alpha \equiv 0.6225, \gamma = 0.873$. The median estimates of StEUT parameters are $\alpha \equiv 0.7144, \sigma = 0.4789$. For every subject the sum of squared errors of the prediction of CPT (StEUT) is explicitly presented in column 4 (7) of table 16. The comparison of columns 4 and 7 reveals that overall CPT and StEUT achieve a very similar fit to the individual choice patterns. However, StEUT explains better the choice decisions of 19 subjects and CPT explains better the choice decisions of 27 subjects. CPT with median parameter estimates α and γ also explains better the aggregate choice pattern across all 80 subjects (*SSE*=0.7682) than does StEUT (*SSE*=0.9732). Thus, for Hey and Orme (1994) study the conclusions drawn from the analysis of the individual and the aggregate choice patterns appear to be qualitatively similar.

Subject #	Cumulative prospect theory			Stochastic expected utility theory		
	alpha	gamma	*SSE*	alpha	sigma	*SSE*
1	0.5533	0.8972	5.3208	0.7184	0.4872	5.8456
4	0.6721	0.8876	4.5633	0.7882	0.4598	4.561
5	0.5678	0.8433	5.1419	0.6695	0.5537	5.2469
6	0.5473	0.8158	3.5151	0.6599	0.558	3.5783
7	0.9137	0.9199	8.4327	1.285	0.3548	8.1172
9	0.629	1.0234	10.0174	0.807	0.405	10.3516
10	0.4535	1.8113	5.4517	0.3852	0.3087	5.9505
11	0.4574	0.8369	2.7523	0.6135	0.5887	2.9329
12	0.442	0.8464	3.7372	0.5997	0.518	4.1199
13	0.2138	1.668	6.2414	0.4552	0.3809	7.2378
14	0.727	0.9167	5.3901	0.8622	0.3981	5.2137
16	0.2398	1.4418	8.471	0.4656	0.4143	10.3592
18	0.5618	0.8843	3.3803	0.6552	0.4774	3.5025
19	0.5544	0.8654	2.3263	0.6733	0.5049	2.446
20	0.6903	0.9799	8.2491	0.8143	0.3953	8.6262
21	0.6274	0.8249	5.7729	0.7676	0.5793	5.7606
22	0.5266	0.8844	3.7732	0.6246	0.5035	3.9335
23	0.6176	0.8805	3.3674	0.7192	0.4576	3.6012
24	0.7353	0.7971	11.6094	1.1695	0.6382	10.3412
25	0.6247	0.8577	6.0376	0.7635	0.5181	6.2026
28	0.5992	0.7424	5.7855	0.6348	0.659	5.7946
29	0.6036	0.7438	9.581	0.7118	0.7769	9.3522
30	0.6317	0.8807	3.7912	0.7169	0.4629	3.7638
32	0.6108	0.891	5.4559	0.6763	0.4418	5.439
33	0.8648	0.8854	8.4092	1.255	0.4055	7.8472
34	0.6738	0.9924	7.5319	0.8606	0.402	7.7794
35	0.7552	0.5129	6.2454	0.615	0.5538	6.5652
36	0.6723	1.0013	6.4409	0.7549	0.3687	6.7587
38	0.2815	0.9412	5.239	0.4764	0.4398	5.7539
39	0.6589	0.8627	4.088	0.7066	0.4479	3.9757
40	0.7953	0.9553	5.6457	1.0013	0.3869	5.4089
50	0.7273	0.903	8.1461	0.8384	0.4032	7.8715
52	1.0378	0.8266	6.3304	2.3203	0.4737	6.7367
54	0.7852	0.8509	8.1317	1.0728	0.4725	7.0625
56	0.597	0.8281	5.4686	0.6893	0.5265	5.6743
58	0.8268	0.7764	6.6511	1.32	0.672	6.0058
59	0.5782	1.896	8.7727	0.4157	0.2581	10.0663
63	0.6978	0.7534	8.4239	0.8517	0.6981	7.9843
66	0.6956	0.8346	4.7077	0.8046	0.5175	4.5988
68	0.6816	0.8457	5.2958	0.8269	0.5168	5.2613
69	0.0809	0.6553	6.7796	0.3437	0.5419	8.1138
70	0.122	2.2811	8.0005	0.4171	0.4655	10.1323
75	0.6202	0.6786	6.1171	0.6956	0.9342	5.7462
77	0.5148	0.6913	6.3225	0.6261	0.7232	6.4113
78	0.5958	0.9058	4.2415	0.6843	0.4803	4.3726
79	0.8009	0.813	7.2907	1.0804	0.554	7.1666

Table 16 Hey and Orme (1994) dataset: the best fitting parameters of CPT and StEUT estimated individually for every subject and the sum of squared errors of the prediction of CPT and StEUT

4.11. The effect of monetary incentives

To conclude this reexamination of the experimental evidence, it is interesting to compare the best fitting parameters of StEUT across all ten studies that employ different incentive schemes to motivate the subjects. Table 17 lists all ten experimental studies that are examined in this paper. For every study, table 17 presents the type of incentives used in the experiment and the best fitting parameters of StEUT. With the exception of Camerer (1992), all studies that use hypothetical incentives have a much higher best fitting parameter σ of StEUT. This suggests that parameter σ can be interpreted as the standard deviation of random errors that are not lottery-specific but rather experiment-specific. The coefficient σ is lower in the experiments that use real incentives and in the experiments that involve negative outcomes.

Experimental study	Type of incentives	Best fitting parameters of StEUT	
		Power of utility function $\alpha(\beta)$	Standard deviation of random errors σ
Tversky and Kahneman (1992)	hypothetical	0.7750 (0.7621)	0.7711 0.6075
Gonzalez and Wu (1999)	hypothetical + auction	0.4416	1.4028
Wu and Gonzalez (1996)	hypothetical	0.1720	0.8185
Camerer and Ho (1994)	a randomly chosen subject plays lottery	0.5215	0.1243
Bernasconi (1994)	random lottery incentive scheme	0.2094	0.2766
Camerer (1992)	hypothetical	0.5871 0.9123 (0.5182)	0.0868 0.0914 0.2299
Camerer (1989)	hypothetical	0.3037	0.4816
	random lottery incentive scheme	0.6830 (0.6207)	0.2897 0.2252
Conlisk (1989)	hypothetical	0.5049	1.8580
Loomes and Sugden (1998)	random lottery incentive scheme	0.3513	0.1382
Hey and Orme (1994)	random lottery incentive scheme	0.7144	0.4789

Table 17 Summary of ten reexamined experimental studies: type of incentives and the best fitting parameters of StEUT

5. Conclusions

A large amount of the empirical evidence on individual decision making under risk can be explained by a simple model—stochastic expected utility theory (StEUT). According to StEUT, an individual chooses among lotteries to maximize their expected utility distorted by random errors. Four assumptions about the structure of an error term are the building blocks of StEUT:

1. An error term is additive on the utility scale with utility being defined over changes in wealth rather than absolute wealth levels.

2. An error is drawn from the normal distribution with zero mean which is truncated so that the internality axiom holds, i.e. the stochastic utility of a lottery cannot be lower than the utility of the lowest possible outcome and it cannot be higher than the utility of the highest possible outcome.

3. The standard deviation of an error is higher for lotteries with a wider range of possible outcomes.

4. the standard deviation of an error converges to zero for lotteries converging to a degenerate lottery, i.e. there is no error in a choice between "sure things".

Four assumptions of StEUT are appealing intuitively, and they are sufficient to explain all major empirical facts such as the Allais paradox and the fourfold pattern of risk attitudes. The most important assumption is probably the truncation of the distribution of an error term. It implies that the lotteries whose expected utility is close to the utility of the lowest possible outcome are systematically overvalued and the lotteries whose expected utility is close to the utility of the highest possible outcome are systematically undervalued.

StEUT appears to be a very descriptive model. The reexamination of experimental data from ten well-known studies reveals that StEUT explains the observed choice patterns at least as good as does CPT. This result suggests that a descriptive decision theory can be constructed by modeling the structure of an error term rather than by developing the deterministic non-expected utility theories. However, StEUT does not explain satisfactorily all available experimental evidence. First of all, individuals value the lotteries whose expected utility is close to the utility of the lowest possible outcome significantly above the prediction of StEUT (e.g. the upper left region of table 3, problems #9 and #12 in table 10). This problem is even more severe for CPT. Neilson and Stowe (2002) notice that, despite popular belief, the conventional parameterizations of CPT do not explain gambling on unlikely gains (purchase of lottery tickets).

Second, StEUT does not explain violation of betweenness when the modal choice pattern is not consistent with the betwenness axiom (e.g. problem #11 in table 5, problems #7, #12, #15 and #17 in table 6). CPT does not explain this phenomenon either despite the fact that CPT is able to predict such violations theoretically (e.g. Camerer and Ho, 1994). Third, StEUT predicts too many violations of dominance than are actually observed (e.g. problems ## 41-45 in tables 14 and 15). This problem is even more severe for CPT embedded in the logistic stochastic utility. Loomes and Sugden (1998) notice that, in general, a stochastic utility model with an error term additive on the utility scale predicts too many violations of dominance. Thus, potentially, even a better descriptive model than StEUT (and CPT) can be constructed that explains the above mentioned choice patterns. The contribution of the present paper is the demonstration that this hunt for a descriptive decision theory can be more successful with modeling the effect of random errors.

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